Direct Celestial Coordinates Solution for Cosmic Ray Shower Direction

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1 Introduction

I want to take a different approach to solving for the primary angle of the cosmic ray shower, using 3D cartesian coordinates for each of the stations. In our XYZ coordinate system, the Z axis will be parallel to the Earth's rotation axis, the X axis will be the line from the Earth's centre to 0,0 latitude and longitude (i.e. on the equator and the Greenwich meridian). The Y direction with point East (perpendicular to the Z and X axes). These are called *Earth Centred Earth Fixed* coordinates. We choose this system because once we have worked out the direction of the shower, all we have to do to get celestial coordinates is a rotation around the Z axis.

The three stations A, B, C are assumed to have coordinates (a_x, a_y, a_z), (b_x, b_y, b_z) and (c_x, c_y, c_z). These are easy to work out from the geographical latitude,ϕ, longitude East, θ, and the Earth's radius, R. That is:

$$x = (R + \Delta R).cos(\phi).cos(\theta) \quad (1)$$

$$y = (R + \Delta R).cos(\phi).sin(\theta) \quad (2)$$

$$z = (R + \Delta R).sin(\phi) \quad (3)$$

We have corrected for height above the ground by increasing the radius, R by the height ΔR.

We now want to work out a shower direction in this coordinate system represented n the form of a vector $\overline{d}$ of unit length from the origin pointing to the arrival direction of the shower. The unit length means that, which means that: $\overline{d}.\overline{d} = d_x^2 + d_y^2 + d_z^2 = 1$, in terms of the vectors x,y,z components.

Let us assume that the shower is first detected at station A, at time t=0, and subsequently at station B at time t_B and station C at time t_C. See Figure 1 for the geometry, Note that the shower in this diagram is moving bottom left to top right along the line of the grey arrow.

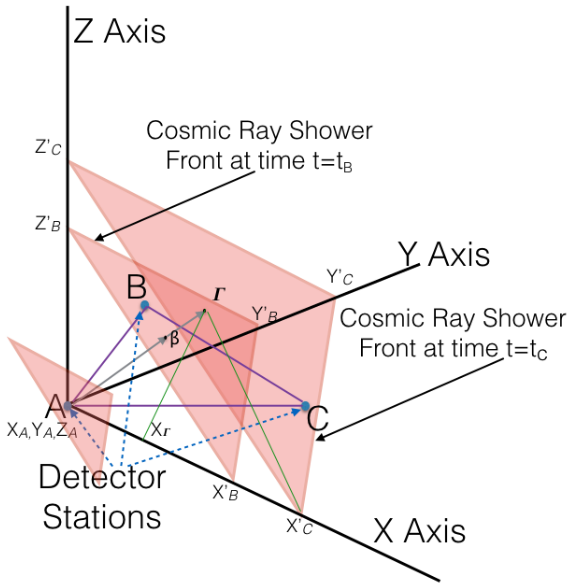

Figure 1: Cosmic Ray Shower Geometry

In order for us to have a valid calculation we have to assume that the cosmic ray shower front moves at essentially the speed of light, which we will denote here by the conventional symbol used by physicists, c. Given the accuracy with which time differences can be measured this will be sufficiently good. The shower then moves as a narrow *front* - almost like a circular plate touching first A, then B then C. So, the distance Aβ in the diagram will be $c.t_B$ and $A\Gamma$ will be $c.t_C$. How do we express these distances in terms of the angle at which the shower is moving?

2 Equation of Motion of a CR Shower Front

The general equation of a family of parallel 2D planes can be expressed as:

$$p.x + q.y + r.z = s \tag{4}$$

where the values of p,q and r stay the same and the value of s changes to distinguish the planes. Note that there is a certain amount of ambiguity in these values: we could divide through the equation by a constant and still get effectively the same set of planes. This will all drop out in the wash later, since all we are interesting in is the ratios of these values to determine the direction of the shower.

At this point we might note that solving for direction with a four-detector coincidence might be easier than handling the three detector case. With just three detectors there is an inevitable ambiguity in the values of p:q:r. You will always get at least two solutions for the ratios because the same set of time differences in the shower arrival times could be produced by a shower approaching from below the detector arrays as a shower approaching from above. Hence, we expect to end up with an equation with two solutions (a quadratic) for the three detector case. In the four detector case, we have more equations and as long as the forth detector is not in the same plane as the other three there is a unique solution that can be found by a relatively straightforward solution of four simultaneous equations. We will look at this case elsewhere.

Hence, when we substitute the three detector locations into the equations we get:

$$\begin{aligned} s_A &= p.x_A + q.y_A + r.z_A && (5) \\ s_B &= p.x_B + q.y_B + r.z_B && (6) \\ s_C &= p.x_C + q.y_C + r.z_C. && (7) \end{aligned}$$

Note that the (x_B, y_B, z_B) and (x_C, y_C, z_C) are NOT the same as the (x'_B, y'_B, z'_B) or (x'_C, y'_C, z'_C) marked on the diagrams at which the front surfaces intersect the axes. These intersection points are related to the orientation of the shower front and the values of p,q and r. So,

$$\begin{aligned} x'_A &= s_A/p & y'_A &= s_A/q & z'_A &= s_A/r && (8) \\ x'_B &= s_B/p & y'_B &= s_B/q & z'_B &= s_B/r && (9) \\ x'_C &= s_C/p & y'_C &= s_C/q & z'_C &= s_C/r. && (10) \end{aligned}$$

What we want to do now is to derive the distances $A\beta$and $A\Gamma$ in terms of these parameters, and then we can solve for p:q:r and get the direction of the shower,

Now let us think about the point, Γ, which is in the plane of the shower front at time t_C and the line to station A is a perpendicular to the shower front, and consider the triangles $A\Gamma X'_C$ and $A\Gamma\Gamma_C$. These are both right-angle triangles and are clearly similar, so:

$$\frac{A\Gamma}{x_\Gamma - x_A} = \frac{x'_C - x_A}{A\Gamma} \implies x_\Gamma - x_A = \frac{A\Gamma^2}{x'_C - x_A} \implies x_\Gamma - x_A = \frac{p.A\Gamma^2}{s_C - s_A} \tag{11}$$

Since, from above, $X'_C = s_C/p$ (and there are exact equivalent derivations of the Y and Z directions) we can immediately write:

$$x_\Gamma - x_A = \frac{A\Gamma^2.p}{s_C - s_A}, y_\Gamma - y_A = \frac{A\Gamma^2.q}{s_C - s_A}, z_\Gamma - z_A = \frac{A\Gamma^2.r}{s_C - s_A}. \tag{12}$$

Adding the square of these together must also give the squared distance from A to Γ by Pythagoras:

$$A\Gamma^2 = A\Gamma^4.(p^2 + q^2 + r^2)/(s_C - s_A)^2 \tag{13}$$

or:

$$A\Gamma^2 = (s_C - s_A)^2/(p^2 + q^2 + r^2) \implies A\Gamma = (s_C - s_A)/\sqrt{p^2 + q^2 + r^2}. \tag{14}$$

We know how to find s_C, we just substitute from equation 7, which represents the shower front at time t_C, to get:

$$A\Gamma = \frac{p.(x_C - x_A) + q.(y_C - y_A) + r.(z_C - z_A)}{\sqrt{p^2 + q^2 + r^2}}. \tag{15}$$

This is the 3D generalisation of the equations 4.3 and 4.4 in the *Primary Particle Angle* report by Koos Kortland.

We could just as well have worked with the coordinates for detector B instead of detector C to get:

$$A\beta = \frac{p.(x_B - x_A) + q.(y_B - y_A) + r.(z_B - z_A)}{\sqrt{p^2 + q^2 + r^2}}. \tag{16}$$

It is worth noting at this point that because we are going to solve for angles we only need ratios of p:q:r. So, for example, the angle, ϕ, between the direction of the shower (along the line $A\beta$ and $A\Gamma$) can be found from

$$cos(\phi) = \frac{z_C - z_A}{A\Gamma} = \frac{A\Gamma.r}{s_C}. \tag{17}$$

Working through the substitutions using the above relations quickly gives:

$$\frac{z_C - z_A}{A\Gamma} = \frac{r}{\sqrt{p^2 + q^2 + r^2}} \tag{18}$$

or, dividing the left hand side above and below by p we get:

$$cos(\phi) = \frac{z_C - z_A}{A\Gamma} = \frac{\left(\frac{r}{p}\right)}{\sqrt{1 + \left(\frac{q}{p}\right)^2 + \left(\frac{r}{p}\right)^2}}. \tag{19}$$

The azimuth angle, θ, (the angle with the X axis of the projection of $A\Gamma$ on the XY plane) is derivable from equation 12 as:

$$tan(\theta) = \left(\frac{y_C - y_A}{x_C - x_A}\right) = \left(\frac{A\Gamma^2.q}{s_C - s_A}\right)\left(\frac{s_C - s_A}{A\Gamma^2.p}\right) = \frac{q}{p}. \tag{20}$$

In order to solve for the p:q:r ratios we now just state equations 15 and 16 for the three planes at time $t = 0$, $t = t_B$ and $t = t_C$, substituting for the distances $A\Gamma$ and $A\beta$ with the respective shower flight times:

$$c.t_B = \frac{p.(x_B - x_A) + q.(y_B - y_A) + r.(z_B - z_A)}{\sqrt{p^2 + q^2 + r^2}} \tag{21}$$

$$c.t_C = \frac{p.(x_C - x_A) + q.(y_C - y_A) + r.(z_C - z_A)}{\sqrt{p^2 + q^2 + r^2}} \tag{22}$$

We now have the task of solving for p:q,r from these three equations in three unknowns. Since we only need the ratios q/p and r/p we have just two unknowns for the two equation, so a solution is feasible. Since we only ever have differences in the X,Y,Z coordinates we can also replace $(x_B - x_A)$ by ΔX_B etc. It is better, therefore to rewrite equations 21 and 22 as:

$$c.\sqrt{1 + \left(\frac{q}{p}\right)^2 + \left(\frac{r}{p}\right)^2} = \frac{\Delta x_B + \left(\frac{q}{p}\right).\Delta y_B + \left(\frac{r}{p}\right).\Delta z_B}{t_B} \tag{23}$$

$$c.\sqrt{1 + \left(\frac{q}{p}\right)^2 + \left(\frac{r}{p}\right)^2} = \frac{\Delta x_C + \left(\frac{q}{p}\right).\Delta y_C + \left(\frac{r}{p}\right).\Delta z_C}{t_C}. \tag{24}$$

To simplify future manipulations I am going to replace q/p by Q and r/p by R:

$$c.\sqrt{1 + Q^2 + R^2} = \frac{\Delta x_B + Q.\Delta y_B + R.\Delta z_B}{t_B} \tag{25}$$

$$c.\sqrt{1 + Q^2 + R^2} = \frac{\Delta x_C + Q.\Delta y_C + R.\Delta z_C}{t_C}. \tag{26}$$

Here we have two equations in two unknowns Q and R. There ought to be a solution - though the square root looks like it is going to make things a little messy.

3 Solutions

From equations 23 and 34 subtract to get the simpler form:

$$0 = \left(\frac{\Delta x_C}{t_C} - \frac{\Delta x_B}{t_B}\right) + Q.\left(\frac{\Delta y_C}{t_C} - \frac{\Delta y_B}{t_B}\right) + R.\left(\frac{\Delta z_C}{t_C} - \frac{\Delta z_B}{t_B}\right) \tag{27}$$

which we could also write as:

$$Q = -R.\frac{(\Delta z_C.t_B - \Delta z_B.t_C)}{(\Delta y_C.t_B - \Delta y_B.t_C)} - \frac{(\Delta x_C.t_B - \Delta x_B.t_C)}{(\Delta y_C.t_B - \Delta y_B.t_C)}. \tag{28}$$

Again for the sake of simplicity of substitution later on, I will define two coefficients consisting entirely of known quantities:

$$U = -\frac{(\Delta z_C.t_B - \Delta z_B.t_C)}{(\Delta y_C.t_B - \Delta y_B.t_C)} \tag{29}$$

$$V = -\frac{(\Delta x_C.t_B - \Delta x_B.t_C)}{(\Delta y_C.t_B - \Delta y_B.t_C)}. \tag{30}$$

When programming this solution we will be able to declare set the values of variables called U and V from these equations. Note, however, because these are ratios we have to guard against the possibility that the denominator may be close to zero. In practice, we can usually recast the relationship between Q and R (equation 28) so that a numerator goes to zero rather than a denominator. These situations alway occur in special cases, where we do not need to go on to solve the quadratic equation below: we can deduce the p:q:r ratios directly - see the ‘Sanity Check’ in Section 3.1 below.

So, we can now write:

$$Q = R.U + V. \tag{31}$$

3.1 Sanity Check 1

We can do a sanity check on this equation. Let us assume that all the detectors are in the XY plane ($\Delta z_{ABC} = 0$), and also that $\Delta y_B = 0$ so station B is on the X axis and $\Delta x_C =$ so stations C is on the Y axis, and for convenience $\Delta x_B = \Delta y_C$ (so we have an isoscolese triangle). This means that U=0, and

$$Q = V = \frac{t_C}{t_B} \tag{32}$$

This is fine! If $t_C = t_B$ the shower is approaching along a line equally spaced from the X and Y axes, and by substituting $Q = q/p = 1$ back into equation 20 we would calculate that $\theta = 45^0$. Similarly, if $t_C = 0$ it means that the shower is moving along the X axis and we would calculate $\theta = 0$, while if $t_B = 0$ we would calculate $\theta = 90^0$. So far it looks good.

3.2 Continuing the solution..

There now does not seem to be any way forward in the solution for R=(r/p) other than to square out equation 25 or 26 and back substituting from 28 and then solving the quadratic in R=(r/p). Here we will follow through from equation 26, but the equivalent substitution in equation 25 obviously goes through in the same way and we will be able to write down the answer immediately.

$$\begin{aligned}1 + Q^2 + R^2 = &\left(\frac{\Delta x_C}{c.t_C}\right)^2 + Q.^2\left(\frac{\Delta y_C}{c.t_C}\right)^2 + R^2\left(\frac{\Delta z_C}{c.t_C}\right)^2 \\ &+ 2.Q.\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta y_C}{c.t_C}\right) \\ &+ 2.R.\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right) \\ &+ 2.Q.R.\left(\frac{\Delta y_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right).\end{aligned} \tag{33}$$

Collecting terms in R:

$$\begin{aligned}&R^2\left[1 - \left(\frac{\Delta z_C}{c.t_C}\right)^2\right] - 2R.\left[\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right)\right] \\ &+Q^2\left[1 - \left(\frac{\Delta y_C}{c.t_C}\right)^2\right] - 2Q.\left[R\left(\frac{\Delta y_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right) + \left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta y_C}{c.t_C}\right)\right] \\ &+\left[1 - \left(\frac{\Delta x_C}{c.t_C}\right)^2\right] = 0\end{aligned} \tag{34}$$

Now we have to substitute for Q from equations 31:

$$\begin{aligned}&R^2\left[1 - \left(\frac{\Delta z_C}{c.t_C}\right)^2\right] - 2.R.\left[\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right)\right] \\ &+(U^2R^2 + 2UVR + V^2).\left[1 - \left(\frac{\Delta y_C}{c.t_C}\right)^2\right] \\ &-2(U.R + V).\left[R\left(\frac{\Delta y_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right) + \left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta y_C}{c.t_C}\right)\right] \\ &+\left[1 - \left(\frac{\Delta x_C}{c.t_C}\right)^2\right] = 0\end{aligned} \tag{35}$$

Collecting terms in R^2 and R, we get:

$$R^2\left[1-\left(\frac{\Delta z_C}{c.t_C}\right)^2+U^2\left(1-\left(\frac{\Delta y_C}{c.t_C}\right)^2\right)-2U\left(\frac{\Delta y_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right)\right]$$
$$+2R\left[UV\left[1-\left(\frac{\Delta y_C}{c.t_C}\right)^2\right]-V\left(\frac{\Delta y_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right)-U\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta y_C}{c.t_C}\right)-\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right)\right]$$
$$+V^2\left[1-\left(\frac{\Delta y_C}{c.t_C}\right)^2\right]-2V\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta y_C}{c.t_C}\right)+\left[1-\left(\frac{\Delta x_C}{c.t_C}\right)^2\right]$$
$$=0 \tag{36}$$

Since we will be calculation the answers using software, there is now little point in writing out the explicit quadratic solution. In practice, we would use the above equation to define values for variables a,b,c in the quadratic form Ax^2+Bx+C and then substitute these into $x=(-B\pm\sqrt{B^2-4AC})/2A$, with:

$$A=\left[1-\left(\frac{\Delta z_C}{c.t_C}\right)^2+U^2\left(1-\left(\frac{\Delta y_C}{c.t_C}\right)^2\right)-2U\left(\frac{\Delta y_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right)\right] \tag{37}$$

$$B=2\left[UV\left[1-\left(\frac{\Delta y_C}{c.t_C}\right)^2\right]-V\left(\frac{\Delta y_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right)-U\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta y_C}{c.t_C}\right)-\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta z_C}{c.t_C}\right)\right] \tag{38}$$

$$C=V^2\left[1-\left(\frac{\Delta y_C}{c.t_C}\right)^2\right]-2V\left(\frac{\Delta x_C}{c.t_C}\right)\left(\frac{\Delta y_C}{c.t_C}\right)+\left[1-\left(\frac{\Delta x_C}{c.t_C}\right)^2\right] \tag{39}$$

For convenience of programming we can pre-compute variables

$$X_C=\frac{\Delta x_C}{c.t_C},Y_C=\frac{\Delta y_C}{c.t_C},Z_C=\frac{\Delta z_C}{c.t_C} \tag{40}$$

and simplify the expressions to get the more convenient forms

$$A=\left[1+U^2-(Z_C+UY_C)^2\right] \tag{41}$$

$$B=2UV-2\left[UV.Y_C^2+VY_CZ_C+UX_CY_C+X_CZ_C\right] \tag{42}$$

$$C=1+V^2-[VY_C+X_C]^2 \tag{43}$$

If we had chosen to substitute for Q in equation 25 we would obviously get and equivalent set of results, where every occurence of a term relating the detector C is replaced by the equivalent term for detector B. These are likely to give more accurate results when $t_B > t_C$ (especially if t_C is very close to zero).

$$A = \left[1 + U^2 - (Z_B + UY_B)^2\right] \tag{44}$$

$$B = 2UV - 2\left[UV.Y_B^2 + VY_BZ_B + UX_BY_B + X_BZ_B\right] \tag{45}$$

$$C = 1 + V^2 - [VY_B + X_B]^2 \tag{46}$$

Use this alternative formulation when $t_B > t_C$, and both are greater than 0. (If both $t_B = t_C = 0$, then we immediately deduce that $\phi = 0$ and θ is undefined - i.e. can take any value (just as longitude is undefined at the Nortth Pole).

Having solved for R, we can get Q from equation 31, and then solve for the declination azimuth angles ϕ and θ using equations 19 and 20.

3.3 Sanity Check 2

Let us do another sanity check, again using our stations that all have $\Delta z_{ABC} = 0$, $\Delta x_C = 0$, $\Delta y_B = 0$ and $\Delta x_B = \Delta y_C$ as before, and again we will set the shower up so that $Q = q/p = 1$ (i.e. with $t_C = t_B$) coming equally between the X and Y axes), though this time we will go on to consider the cases where $R = r/p \neq 0$. Referring back to the equations defining U and V (29 and 30) we immediately get U=0 and $V = \Delta x_B.t_C/\Delta y_C.t_B = t_C/t_B = 1$. The equation 36 for R reduces to:

$$R^2 = \left(\frac{\Delta y_C}{c.t_C}\right)^2 - 2 \tag{47}$$

Note that $c.t_C$ can never be larger than $y_C/\sqrt{2}$ for a real coincidence detection because of the way we have set up the geometry so this expression is always positive. When $t_C = 0$ (that is the shower is moving pretty much along the Z axis) the value of R goes to infinity. Now look at equation 19: both the top and the bottom will be dominated by the R terms and the ratio will tend to $R/\sqrt{R^2}$ which tends to 1, which results in $cos(\phi) = 1 \implies \phi = 0$ as we would expect. On the other hand, when the shower is moving in the XY plane, $c.t_C = y_C/\sqrt{2}$, so R = 0, and from equation 19, $cos(\phi) = 0 \implies \phi = \pi/2$, which again is what we would expect.

We can also check what happens when we put the B and C stations on the X and Z axis respectively, with $\Delta y_B = \Delta y_C = \Delta z_B = 0$ and $\Delta x_B = \Delta z_C$.

Equation 28 at first sight makes p/q undefined, but we can multiply through by $(\Delta yC.t_B = \Delta y_B.t_C)$ and get:

$$R = -\frac{(\Delta x_C.t_B - \Delta x_B.t_C)}{(\Delta z_C.t_B - \Delta z_B.t_C)} = \frac{t_C}{t_B} \tag{48}$$

This looks rather similar to equation 32, which is reassuring, and if we choose to make Q = q/p = 0 (as we are free to do - it means a shower moving in the XZ plane). We can go back to equation 19 and turn it into $cos(\phi) = R/\sqrt{1+R^2}$. Using the trig. identity $cos(x) = 1/\sqrt{1+tan(x)^2}$ then turns this into:

$$tan(\phi) = \frac{1}{R} \tag{49}$$

which is indeed and equivalent form to that we had previously, allowing ϕ to vary between 0 and $\pi/2$ as we might expect.

I think that this suggests that the relations above are all OK, and I get the same answer when I use a computer algebra package, so we are probably good to go.

When programming the expression there are obviously potential pitfalls associated with dividing by zero in certain shower+detector array geometries. (For example, when times such as t_B and t_C go top zero.) One has to trap these special cases before trying to solve the above quadratic and make the correct angle deduction immediately.

It would also be a good exercise to understand the sensitivity of the solutions to small timing errors.

Coincidences are recorded with respect to GPS time derived from satellite signals. Although the GPS clocks are extremely accurate (within nano-seconds) there signals have to propagate to the detector stations through the atmosphere which very slightly slows down radio waves. The propagation delay is unfortunately rather variable, so in extreme cases the GPS time may be in error by up to 100 nano-seconds. Sophisticated GPS receivers, such as those used by the military, commercial ships and aircraft, and professional surveyors can correct to some extent for this error. We do not have this type of receiver.

It would be interesting and useful to take real data and vary some of the time delays by, say 10-20 nano-seconds and see what effect this has on the reported directional solutions. This would tell you the extent to which you can believe the accuracy of the directions you calculate.

4 Solving for Right Ascension and Declination

See figure 2 on page 11 taken from Unsöld (Unsöld 1969). The angles we have calculated, ϕ and θ are not directly the celestial coordinates. We defined the angle ϕ as the angle made by the shower direction with the rotation axis of the Earth, but the astronomical *declination*, usually denoted by δ, is defined as the angle with the equatorial plane, so:

$$\delta = \frac{\pi}{2} - \phi. \tag{50}$$

The angle θ is also *not* the astronomical *right ascension* coordinate (usually denoted by the symbol α) because our xyz coordinate system is rotating with the Earth, and the projected x-axis sweeps round the Earth's equatorial plane projected onto the sky once every sidereal day.

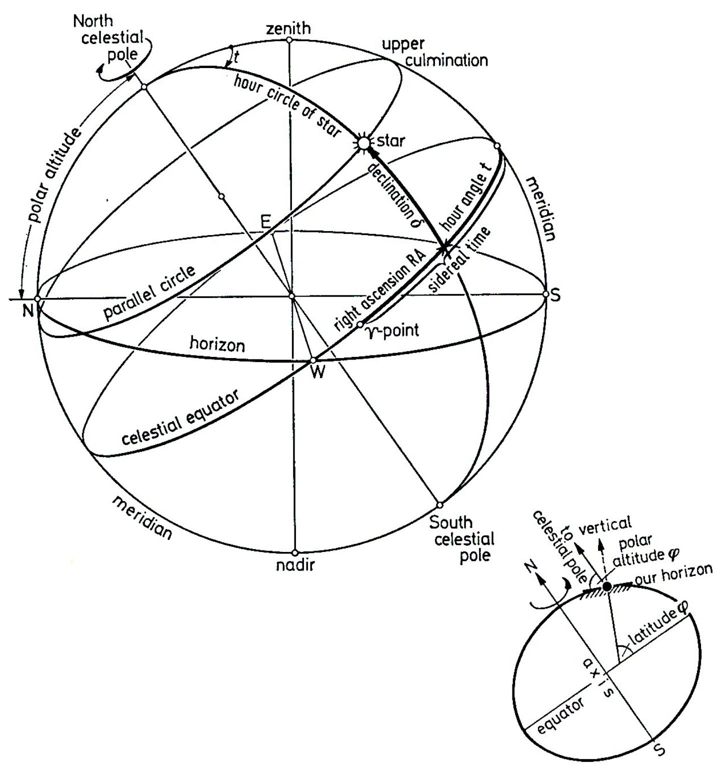

Figure 2: Celestial Coordinates.

The next bit of reasoning is basically straightforward - but it is easy to get confused with definitions and use the wrong signs. Here are the definitions:

North Celestial Pole is the projection of the Earth's rotation axis through its North Pole onto the sky. Observationally, in the northern hemisphere, all the starts appear to rotate around this point (which is approximately marked by Polaris, the pole star).

Zenith is the point directly over your head.

Meridian is the great circle on the sky running from the north celestial pole, though the meridian, down to due South. If you stand looking due South and move your head up and down, your point of view is moving up and down the meridian.

Hour Circle of an astronomical object is the great circle on the sky running through the north celestial pole down through the position of the object of interest.

Hour Angle (HA) is the angle between the meridian and the hour circle of the object. It can be measured in degrees, or hours, minutes and seconds. (The use of time as a measurement is very convenient when doing astronomical observations because the hour angle is just the amount of time that has passed since the object rotated through the meridian.) Since astronomical object appear to move from East to West on the sky, hour angle increases in the westerly direction.

Vernal Equinox also known as the *First Point of Aries*, denoted by the symbol ♈, is the point on the Celestial Equator from which *right ascension* coordinates are measured. (It is actually defined as the point where the Sun crosses the Celestial Equator in the Spring.)

Right ascension, usually denoted by the symbol α, is conventionally measured in hours minutes and seconds (and this is always used in celestial catalogues) but is easily converted to degrees, since 360^o is equivalent to 24 hours. Right ascension increases going round to the east (or looking down from the north pole it increases going anticlockwise). The First Point of Aries crosses the meridian when the local sidereal time (LST) is 0 hours. If we wait until the sidereal time on our astronomical clock equals the right ascension of our object then we will find the object right on the meridian.

Hence it is possible to deduce the following simple relationship:

$$\alpha = LST - HourAngle \tag{51}$$

We have actually defined our xyz coordinate system as if we were standing on the Greenwich Meridian (the line of 0 longitude) since we required our x axis to run from the centre of the Earth through the point of 0 longitude, 0 latitude on the equation. Even though many of our detectors are not actually on this meridian, everything refers back to it. Hence, our local sidereal time is the same as Greenwich Mean Sidereal Time

At this point it is easiest to look up standard formula for Greenwich Mean Sidereal Time (in units of hours), as used by professional astronomers. The formula is very accurate because it takes account of phenomena such as the (very) gradually slowing rotation rate of the Earth.

$$GMST = (18.697374558 + 24.06570982441908D) modulus(24) \tag{52}$$

where D is the number of days (including fractions of a day) since 12 Noon (Universal Time) on 1st January in the year 2000. This is known as the *Julian Day Number* with respect to the year 2000 epoch[1]. The $modulus(24)$ means divide by 24 and keep only the fractional part of the result, it reduces the GMST value to the range 0...24. (That is, we take out any whole multiples of 24 from the answer.)

Various sources on the Web, including the Wikipedia *Julian Day* page, provide moderately intricate algorithms for determining the Julian Day from the calendar date, but for single observations it often easiest just to look up

[1]Beware! If you come across the term Julian Day be careful that you get the right epoch, that is, the point from which days are counted. There are at least eight other epoch definitions used at various times by various subgroups of the astronomy community. The original definition of Julian Days used noon on January 1st 4713BC since it is the starting point of a number of astronomical cycles and also before any historical date associated with an astronomical observation. Using Julian Days to log astronomical observations avoids difficulties with working out long time intervals over periods including leap years and so on. The Julian Day starts at noon because until recently most astronomy took place at night and it was inconvenient to have a single series of observations on the same night being logged as on two different days. This reasoning is now less compelling with instruments such as radio telescopes and satellite observatories that work 24 hours a day, but these days computers keep things straight.

The trouble with the original Julian Day definition is that the numbers become large. For example the Julian Day number for 12:00 UT on January 1, 2000, was 2,451,545. Hence, it is often more convenient when tabulating data to refer Julian Days to a later epoch (such as the start of the new millennium). As explained earlier there are many possible choices found in the astronomical literature. You need to check which one is in use, and a well reported research paper will always make it clear.

one of the on-line conversion utilities such as, for example, from the US Naval Observatory.

Hence if we know the the Julian Day and the hour angle we can find the right ascension. We have calculated a value of θ, which is the angle going eastwards from the x axis to the hour circle of our shower direction. The hour angle is measured positive going westwards, so the hour angle is just the negative of this value.

References

Unsöld, A. (1969), *The New Cosmos*, Springer.