Can You Navigate the Solar System with a Light Sail?

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1 Energy, Energy, Energy, Force, Force, Force

The hardest and most expensive part of spaceflight is getting from the Earth's surface into orbit. (The second hardest is probably getting back down again!) This is an issue of *potential wells*: a kilogram orbiting in space has a lot more *potential energy* than one at the Earth's surface. It also has to keep moving in order to stay in orbit and that means that there is also *kinetic energy*.

Once we are well out of the Earth's potential well - say, somewhere like the Geostationary Orbit (GSO) height the energy cost of getting anywhere else in the Solar System is relatively low.

Furthermore, although we have to exert very high forces to lift a massive rocket against the acceleration due to the Earth's surface gravity and also to accelerate it to orbital velocity, once we are in orbit the forces required are much lower - providing we do not have to do anything very quickly.

At present, however, the energy still has to come from burning rocket fuel and all that fuel still has to be lifted from the Earth's surface. This process is very energy inefficient because we have to burn the rocket fuel all the way from the Earth's surface until we get into low-Earth orbit (LEO) , so the fuel that we burn later in the flight has itself to be lifted out of the gravitational potential well and also has to be given kinetic energy, as part of the rockets motion.

It turns out that for every kilogram of fuel we may want to burn after leaving GSO we probably have to burn about twenty kilograms or more getting it there.

If we want to have a serious industrial presence in space, for example to mine asteroids for rare and essential elements that are running out on Earth we need a better way—otherwise it will all get too expensive.

The fuel efficiency of a rocket motor depends on the amount of momentum you can give to each kilogram of material in the exhaust plume. If you can can eject the exhaust faster you use less fuel to get the same acceleration. The exhaust velocities of chemical propellants are limited by the amount of chemical energy contained in each kilogram of fuel: the higher the specific energy content, the higher the potential exhaust velocity. At the atomic scale it is a question of releasing energy in a chemical reaction between two molecules which is then used to accelerate the reaction products. Ideally you want reactions that involve big releases of energy, but you also need light molecules so that they heated to high temperatures and accelerated to high velocities. This is one reason why NASA's space shuttle was partly powered by using liquid hydrogen and oxygen as the propellant: it has a high specific energy release and the lightest product molecule. (There are other reasons why engineers often choose *not* to employ this combination, not least because explosive cryogenic liquids are very difficult to handle safely.) All this means that our rocket motors have pretty much reach the theoretical limit of what is possible with chemical propulsion.

The *Tsiolkovsky Rocket Equation* nails down the fundamental relationships that dominates the thinking of every rocket engineer and astro-navitagor:

$$\Delta v = v_e \log\left(\frac{m_0}{m_f}\right) \quad \Rightarrow \quad m_o = m_f.exp\left(\frac{\Delta v}{v_e}\right) \tag{1}$$

where:

- Δv is the change of velocity actually achieved,
- v_e is the exhaust velocity,
- m_0 is the initial mass (including propellant - the 'wet' mass),
- m_f is the final mass after the rocket shuts off. It is sometimes called the 'dry' or empty mass, but the equation still applies if you need to keep fuel in your spacecraft for further manoeuvres. The equation relates the ''before and 'after' mass of the spacecraft to the velocity change.

This basically says that if we have to work with a fixed v_e then we are on a hiding to nothing as we try to attain higher and higher velocities (e.g. if we want to send a spacecraft out to Jupiter in a reasonably timescale). You are pouring more and more energy into accelerating more and more fuel just so you can throw it out of the back of the rocket.

This is why nearly all deep space missions use planetary 'sling-shots' to give spacecraft very high velocities. (The astro-navigators design the course so that the spacecraft effectively bounces off the potential well of an approaching planet - fundamentally the same as a using tennis racket to increase the velocity of a ball, except that the we do not actually hit the planet, just get pulled around the back in a close orbit.) Of course, planets do not necessarily line up where and when you want them, so this puts very strong constraints on when it is possible to launch deep space missions, and if you miss a window it may not re-open for a long time.

Getting to the outer planets will always be difficult and time consuming: either we have to take highly convoluted routes involving slingshots off several planets—sometimes bouncing between Venus and Earth several times—or else you have to burn lots of fuel to get the spacecraft directly into a high velocity transfer orbit. It

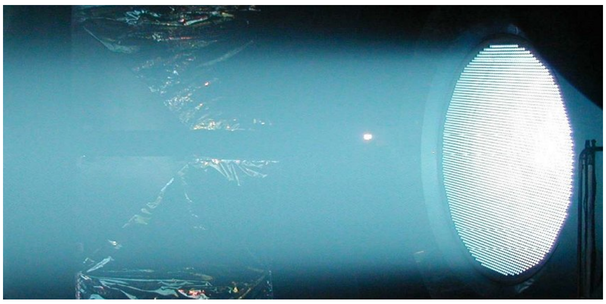

Figure 1: The NEXIS Ion Thruster on test. (Image credit: NASA - public domain material)

will still take a long time to get there because there will be a long coasting phase.

The only other way to get higher velocities for less fuel mass is by using higher exhaust velocities generated by non-chemical methods. There have been design studies for nuclear rockets, but the engineering problems have so far prevented any practical demonstrations.

There are, however, hyper-efficient 'ion-drives' in use today which accelerate charged particles to extremely high exhaust velocities, using electrical energy gathered from solar panels. Ion drives cannot generate the same level of force as chemical rockets, but on the whole they are used in situations where they do not need to. Furthermore, if you can maintain a small acceleration for a long time, you do eventually build up a very high velocity.

Ion drives are very fuel and energy efficient, but they still consume fuel mass even though it no longer has to provide the energy. Ion drives are used today to move communications satellites out to the geostationary orbit and maintain their positions accurately. With such a high efficiency drive these spacecraft can easily carry enough fuel to last throughout a long projected lifetime.

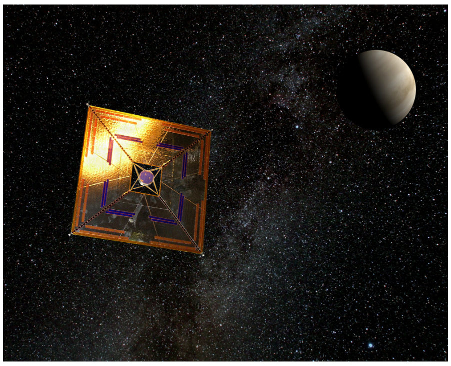

Figure 2: Artist's Impression of the IKAROS Solar Sail Experiment (Image credit: Wikipeida Creative Commons)

For longer spaceflights under continuous acceleration the fuel would eventually run out, so it would be better if we could do without fuel entirely, even if we have to accept even lower accelerations. Given that it always take years to send spacecraft to the outer Solar System it may well be that continuous acceleration even under small accelerations may be competitive with multiple and convoluted slingshots or direct transfer orbits with coasting. Comparing these options will be part of your challenge.

We can do without internal fuel if the source of momentum is external to the craft, and this principle has been used for millennia with sailing craft on the Earth's surface waters. Is it possible to 'sail' across the Solar System?

The Sun, besides being a source of energy (in the form of light) is also a source of momentum, because each photon carries an amount $h\nu$ of energy and also $h\nu/c$ of momentum, where h is Planks constant (6.62607×10^{-34} $J.s$), ν is the frequency of the light, and c is the velocity of light (3×10^{8} ms^{-1}). Hence, if we hold a mirror up to the Sun the light is exerting a very small force which tries to push the mirror away.

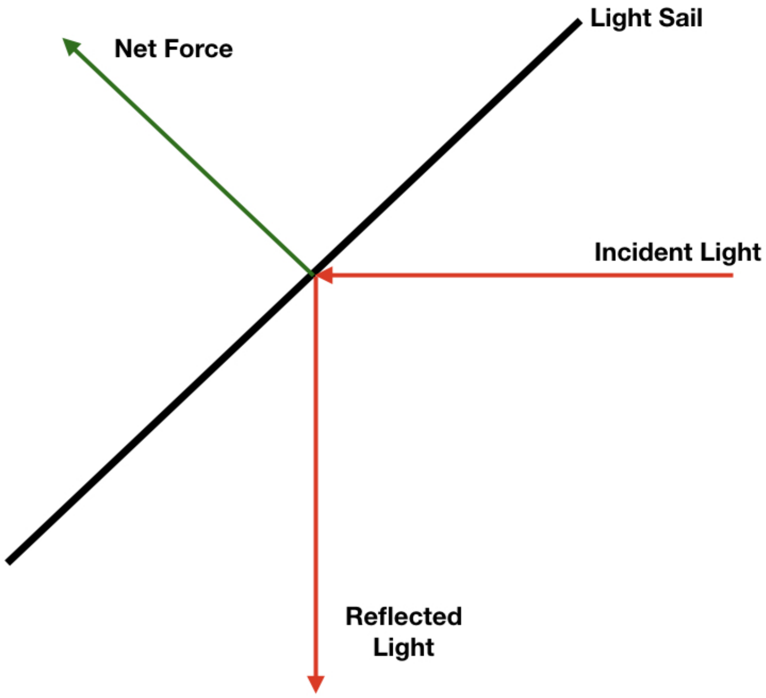

Figure 3: Force on a Solar Sail

This is indeed a very small force (you will need to plug the numbers in to work out the force per unit area) but it does not go away and we now know that over millions of years it can move asteroids around. This is the so-called *Yarkovsky* effect, and the orbital changes it produces over a few years can now be detected using accurate planetary radar measurements. (The asteroid *6489 Golevka* was carefully tracked by the Arecibo planetary radar between 1991 and 1999 and was found to have moved 15km from where its orbit should have placed it.)

So we will now imagine a spacecraft that is able to extend a very large *solar sail* which is preferably highly reflecting, and that it can orient the sail in various directions with respect to the line of sight to the Sun. If we just hold our sail perpendicular to the direction of Sun, nothing very exciting would occur. The force of gravity is an inverse square law and the intensity of light decreases with distance as an inverse square law so the net effect would be seen as a very small but proportionate reduction in the inward force, which would still be an inverse square law. Effectively we would see this as a slight (very slight) reduction in the Sun's gravitational mass. We could, of course, also orientate the sail's surface in line with the Sun's direction and all the photons pass by so again, nothing much would happen.

Interesting things happen, however, when we hold the mirror at an angle - say 45 degrees to the Sun's direction. (See Figure 3.) If photons are reflected back along the orbit we have a component of force tending to *increase* the orbital velocity This tends to push the spacecraft further out from the Sun in a very tightly wound spiral (every orbit just slightly larger than the last). If photos are reflected forward along the orbit direction it would tend to slow the orbital velocity and the spacecraft would spiral slowly towards the Sun. We are going to get you to calculate how long this would take!

The effect of light on asteroids is, in fact, very complicated because they are often irregularly shaped and also rotating. Over millions of years, the light pressure on an irregularly shaped asteroid can make it spin faster and faster. In some cases this seems to go on until the asteroid breaks into two parts—it seems to be the only way to account for the large number of very close binary asteroids which are almost touching as they spin around each other. When light is absorbed on the rough surface of an asteroid there is a small delay before it gets re-emitted and when this effect is combined with the spinning the emitted light is effectively bounced off in a different direction to where it came from. Depending on which direction the asteroid is spinning this can mean it will be either slowed down or speeded up in its orbit and either spiral inward towards the Sun or outwards away from the Sun. The Yarkovsky effect is thought to be the main reason asteroids slowly leak out of the asteroid belt, until eventually some have a close encounter with Jupiter and get flung into the inner Solar System (and occasionally hit the Earth).

We cannot wait millions of years to move spacecraft around the Solar System, so we need to know if engineering constraints allow us to build mirrors big enough to move serious payloads around in space. Of course, increasing the area of such a sail increases the amount of force we experience, but it also increases the spacecraft mass and if the material from which the mirror is made is heavy the we are going nowhere at all quickly. So, we need to think carefully about practical materials and look for stuff that is very strong for its weight so we can make it very thin while still being able to handle the forces.

The next mission to Mars is not going to be powered by a light sail, because the technology is just not yet there, but nor are the challenges so out of our reach that it is just not worth thinking about it at all. We are in a situation where a bit of serious analysis of the possibilities might make a case for directing research effort towards specific technical problems that need to be overcome—and could perhaps be solved with a sufficient amount of focus. One of the engineering issues that has received some attention is how to pack large flat structures into spacecraft for launching and then be able to reliably unfold them in space without tangling, sticking or tearing. This has some surprising connections. Korya Miura, a Japanese astrophysicist and paper-folding enthusiast invented a new method of folding flat sheets[1] to help with packing

[1] I described the *Miura fold* to my textile-artist sister, who was having trouble working out how she should produce a large and relatively stiff textile installation in her workshop and then move it to an exhibition centre using her small car, and she said "I can use that!"

and unpacking large solar arrays from spacecraft. NASA now has engineers who study origami.) See Figure 4 for an example of a Miura Fold[2].

In fact some small and rather low cost spacecraft have already been launched to examine some of the engineering problems. The crowd-funded Lightsail-1 was launched in 2016 by the Planetary Society, and they plan to launch Lightsail-2 in late 2018. (It is already in the launch queue and just waiting for its turn.) Lightsail-2 will attempt to manoeuvre in space while being tracked from Earth using laser rangefinding.

The essence of your project is to understand the nature of the technical challenges, using quantitative calculations as far as possible, and determine just how far away we are from being able to deploy useful solar sails on spacecraft. And...just how far *could* we push this technology? The pressure of sunlight is rather low, but suppose we constructed powerful lasers on the Earth's surface and focussed them on a lightsail. What sorts of accelerations and ultimate velocities might be achieve? Could we even send a spacecraft to another star? Look up the *Breakthrough starshot* project to see how the feasibility of really far-out ideas can be seriously investigated.

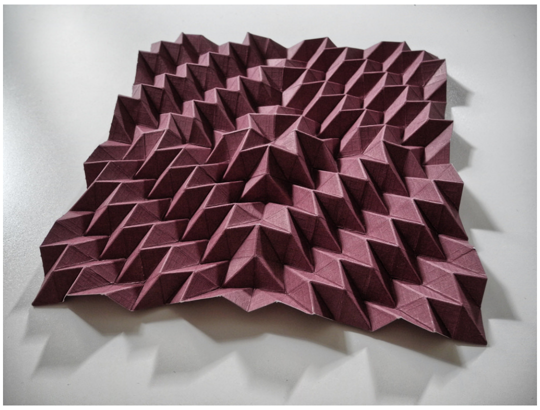

Figure 4: A type of Miura Fold (See footnote for image credit.)

2 Questions, Questions, Questions

The best way to start this type of project is by making a list of questions, outlining some of the major technical issues that need to be addressed.

- What type of force per unit area can be achieve using the pressure of sunlight?
- How can be use this force to navigate a spacecraft in any desired direction (even against the direction of the force)?
- How big can we make a light sail?
- What shape should it be? (You may see images on the web of light sails built like parachutes, connected by tethers from the edge of a flexible circular disk to a trailing spacecraft. Why will this not work? Think about the direction and

[2] See the interesting website https://naturalorigami.wordpress.com/2016/07/18/the-miura-ori-fold/. You might like to see if you can invent your own variations on this fold.

magnitude of force on each element of a such a light sail design. How will the forces distort the sail.

- What materials should we use? You will find that mass per unit area is an important parameter. How small can this be while still retaining enough strength to take the forces?
- Mylar is a material that has been used for current experiments. How thin can this get? How much weight do we add if we make it reflecting with an aluminium layer?
- Do such thin materials suffer from 'stickiness' (like cling film) making them hard to handle without tearing? Can anything be done about this?
- Are there new types of material that might be a better solution? Carbon nanotubes? Graphene? What are their properties? Can they yet be made in sufficiently large quantities?
- Would it be possible to manufacture such large thin sheets actually in space, to avoid the packing/unpacking and stickiness problems?
- How small and light might we make payloads while still allowing them to do useful jobs (e.g. contain enough sensors)?
- What type of orbits would we use to get from near-Earth space to, say, Mars? How do we use a light sail to get out of Earth's orbit? How could you get into a Mars orbit?
- How long might it take to get there?
- How do we come back - if we want to come back?

3 Do it with Numbers!

The essence of a science and engineering feasibility study is that you do the sums.

There is a great deal of speculative descriptive material on the Web. We want to know whether any of the science actually hangs together. The core skill of a practicing professional physicist is turning wishes and speculations into questions that can addressed in a quantitative way—even if the answer consists of phrases such as "probably better than" and "probably no worse than".

As with most physics, the questions can ultimately be addressed by thinking about energy, power, force and strength. The easiest calculation is working out the amount of force on a light-sail. I have essentially given you the answer above. For each photon you divide the energy by the velocity of light to get the momentum, and that still works when the photon come in very large numbers. So, you will need to work out the flux of energy from the Sun per unit area. Look up the total luminosity fo the Sun, the length scales in the Solar System, and then use the inverse square law.

For those of you who are seriously thinking of degrees and subsequent careers in physics or engineering I strongly recommend the following two books, which are available to download in PDF format under the Creative Commons licence.

- *The Art of Insight in Science and Engineering* https://mitpress.mit.edu/books/art-insight-science-and-engineering
- *Street Fighting Mathematics* https://mitpress.mit.edu/books/street-fighting-mathematics

The above publisher's pages both contain an "Open Access" link for downloading the PDF—though just Googling the title is likely to bring up direct links to the PDFs. Both of these books teach the art of finding 'good-enough' answers without having to produce exact solutions to complicated problems, and show you how to understand the essential nature of problems, without loosing yourself in mathematical manipulations.

The essential nature of this problem is that the biggest force in play is gravitation. Hence, to a *first approximation* our spacecraft will be moving in an orbit in which energy and angular momentum are conserved (apart from the small inputs from the light-sail forces). Hence, at all times the motion of the space craft will be very nearly described by an ellipse.

So, in our *second approximation* we might think about how an ellipse can be changed is we feed in new energy and angular momentum slowly from a light-sail force acting, say, along the forward direction of motion or opposite to the direction of motion. There is some basic stuff here involving Newton's Laws and how you estimate energy changes from applied forces. (A circular orbit, for example, might be expected to either slowly increase or decrease in radius.)

We might also get some *order of magnitude* feel for how long journeys might take by ignoring gravitation and simply asking what would happen to a small spacecraft—a kilogram or so—under uniform light-sail acceleration—assume, say, a square kilometre of sail of about 1/10th the thickness of today's thinest Mylar sheets, running n a straight line from here to Mars or Jupiter. You might be surprised to find that it does *not* take an excessively long time, and may not be that much longer than using chemical rockets and long coasting phases.

Appendix A: The Importance of Delta-V

The *Tsiolkovsky Rocket Equation* expresses the relationship between the increment in spacecraft velocity and the proportion of the fuel that is consumed achieving that change:

$$\Delta v = v_e \log\left(\frac{m_0}{m_f}\right) \quad \Rightarrow \quad m_o = m_f.exp\Big(\frac{\Delta v}{v_e}\Big) \tag{2}$$

When using chemical propellants it is nearly always sensible to use fuel in short, high-thrust ‘burns’ because in that way we may be able to avoid carrying fuel out of potential wells.

So, having achieved a circular low-Earth orbit (at some cost in fuel), we may now wish to move to a higher orbit (for example, geostationary orbit). The normal way we might perform this manoeuvre would be to give a momentum kick to the spacecraft with a short period of high thrust that quickly increases its velocity. (We shall assume that this ΔV is acquired in an effectively instantaneous internval.) Since the spacecraft is now moving too fast to maintain the circular orbit it will now be moving in an elliptical orbit with the perigee (the point of closest approach) at the point where the change in velocity occurred. At some point later the spacecraft will be at apogee (the point furthest away from Earth) but having climbed out of the gravitational potential well for some distance is now moving more slowly. It is, of course, now moving too slow to maintain a circular orbit at the apogee distance so starts to ‘fall-back’ towards the Earth.

At the apogee, however, we can stabilise a circular orbit if we perform another burn to deliver just the right ΔV to increase the spacecraft velocity to that required for a circular orbit at this height. It is easy enough to work out the amount of fuel required for this change in orbits. If the first velocity increment is ΔV_1 and the initial mass is m_o while the mass after the first burn is m_1 then:

$$\Delta V_1 = v_e \log\left(\frac{m_0}{m_1}\right) \tag{3}$$

For the second burn we start with mass m_1 and then finish with mass m_2 having incremented velocity by ΔV_2 so,

$$\Delta V_2 = v_e \log\left(\frac{m_1}{m_2}\right) \tag{4}$$

So, the ratio of the initial mass to that after the second burn m_o/m_2 is just given by:

$$\Delta V_1 + \Delta V_2 = v_e\Big(\log\left(\frac{m_o}{m_1}\right) + \log\left(\frac{m_1}{m_2}\right)\Big) = v_e \log\left(\frac{m_o}{m_2}\right) \tag{5}$$

That is, in order to calculate total fuel requirements for several manoeuvres we just add the ΔVs for all the the individual steps, and then use the rocket equation (2)

Many astronavigation problems are described in terms of ΔV - e.g. ”the ΔV required to rendezvous with asteroid XYZ”.

It may be useful to go outline how one might calculate the required ΔVs for a simple situation. (You can follow through with the detail.) See Figure 5 for an illustration of

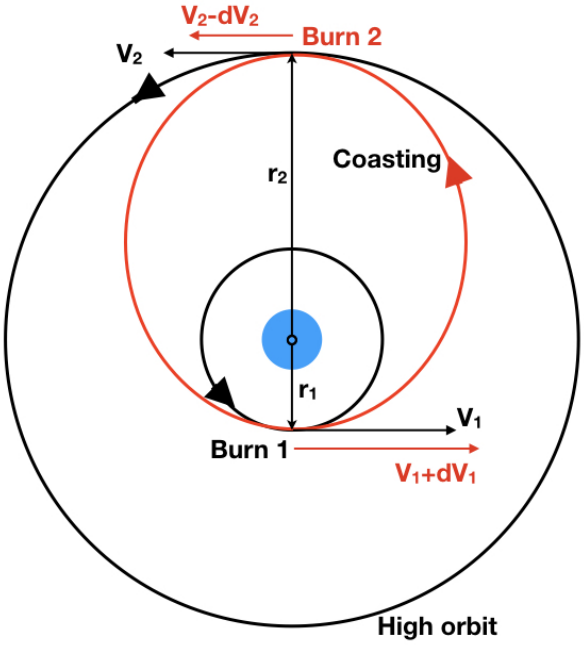

Figure 5: Transfer Orbit

the problem of transferring a spacecraft to a higher orbit (where both orbits are circular). We have two knowns: the radius of the low orbit, r_1 and the radius of the high orbit, r_2, both of which also determine the orbital velocities, V_1 and V_2 from Newton's law of gravitation.

Our task is to calculate the velocity changes required to enter and leave the elliptical transfer orbit. Note that we do *not* need to calculation the exact shape of the ellipse in this simple case because we can rely on two conservation laws: conservation of energy and conservation of angular momentum. The energy at the two burn-points is easy to calculation using the gravitation law as the sum of potential and kinetic energy, which are entirely determined by the position and the velocity. The angular momentum at the perigee and apogee of the ellipse are also easy to calculation because the velocity is exactly perpendicular to the the radius vector at these points. You should now be able to write down two conservation equations, which have two unknowns: dV_1 and dV_2 so you can solve for the unknowns. We then just add the two ΔVs in order to calculate the fuel requirement from equation 2.

Most of astro-navigation using chemical propellants is about gluing together sections of elliptical and circular orbits, connected by ΔV changes. It does, of course, get more complicated when cannot do our velocity changes at the apogee and perigee of the ellipses, but that is why we have computers to do the complicated detail for us.

In reality, when we want to go to Mars or Jupiter, which would required quite large $\Delta V s$ we tend to plan orbits that involve 'slingshots', which uses close orbits around the back of other planets to increase our velocity and energy for free.

ΔV do not help us with fuel calculations for continuous-thrust courses, which are usually less efficient than this quick-burn+coast option. However, while low-thrust propulsion cannon give the almost instantaneous impulses of chemical propellants they burn fuel at a much lower rate and use very much higher exhaust velocities, so they are viable options.

Appendix B: Relevance of Strength of Materials

All the materials that we see around us are held together by atomic bonds which are ultimately based on electromagnetic forces. In principle, the maximum strength of a material depends on the strength of the atomic bonds holding it together. If we want to pull on, say, the two ends of a rod until it breaks in half we have to supply a force that overcomes the combined interatomic forces holding together the planes of atoms in the bar. Real materials always seem to be much weaker than the strength we calculate in this way because they nearly always contain small defects that tend to concentrate stress, so we do not have to break a complete plane of bonds all at once, instead we can break them one by one, like tearing a sheet of paper once a small nick has been made in one edge.

In order to build our Solar Sail we need to be able to make a large reflecting sheet which must be strong enough to carry the forces that will act upon it, but also be as light as possible, so we can get the maximum acceleration from the pressure of the light.

The thinner we make the material the less it is able to carry forces. (It may also be more likely to tear when we are trying to pack it for launch and unpack for spaceflight.) What we need is the material that is strongest for a give weight.

We are used to thinking of materials like steel as very strong, but they are also very dense. Iron is a heavy atom, but inter-atomic forces do not necessarily increase in proportion to an atoms mass. In fact, in terms of its strength/weight ratio, steel is not very good. Aluminium is, of course, a much lighter atom, but its bond strength is also much lower so although its alloys can have higher strength/weight ratios than steel (which is why they are used in aircraft construction) it does not provide a solution to our problem.

We need to look at light atoms that still have strong inter-atomic bonds, and down at the bottom end of the periodic table there are few options, which pretty much resolve down to exotic forms of carbon. We have used carbon fibres in aircraft construction for many years because of its very high strength/weight ratio (which has to be set in epoxy resin because it is otherwise too flexible if you want to build aircraft spars). You could, however, imagine weaving a material out of carbon fibres.

More recently there has been a lot of interest in materials such as carbon nano-tubes and graphene (which I will leave you to research). These promise to be much stronger than conventional carbon fibres and one might imaging using them to make very large and very light sails—if we could produce them in sufficient quantities. That at the moment is the technical barrier: in theory a film of graphene of about the thickness of cling-film could support an elephant, but the largest defect-free samples produced so far are finger-nail sized.

How strong does it need to be to avoid tearing? Let us consider the configuration illustrated in Figure 6, which is not a realistic solar sail design but will allow us to understand some of the issues.

We should imagine that we are looking at a cross section of a sail that extend into and out of the page supported at its edge by two long rods. F_{tot} is the total force due

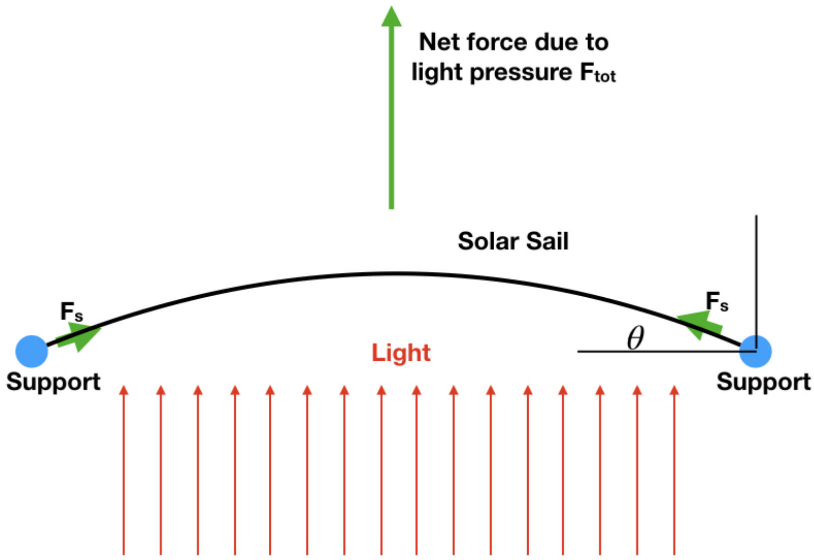

Figure 6: Forces on the edge of a Solar Sail

to light pressure acting on the sail for, say, each meter length in/out of the page. This force has to be transmitted to the rods in the plane of the sail, and creates forces, F_s per unit length at each edge, where the sail makes and angle θ with the plane of the sail.

Using a standard resolution of forces we immediately see that:

$$F_{tot} = 2.F_s.sin(\theta) \tag{6}$$

So, the smaller we make θ (that is, the tighter the sail) the greater the in-sail force F_s that must balance the total forwards force F_{tot}. Of course, the bigger we make θ the more slack there is in the sail and the heavier it will be. (There is also a loss of forward thrust because light is not reflected back along the track.)

Hence, we cannot make a very tight sail, because the forces would tear it apart. There must be an optimum degree of 'slackness' which keeps the forces manageable, but also keeps down the mass of the sail. The engineering team will do many calculations to determine just how thin they can make the sail and still be able to resist the in-sail tensions, and will consider how to determine the optimum amount of slack,

In a real sail design you would also need to take account of the number, weight and strength of the bracing struts to which the sail is secured. Is it better to use many weak struts or a fewer, larger heavier ones? Why do we have to use stiff struts? Why cannot we use a parachute type configuration with flexible tethers? (Explain why the sail could not hold its shape under the light-pressure.)

Incidentally, would the sail end up in the shape that I have drawn when under the pressure of light? You might like to think about a stable shape in which the force due to light at each point on the sail can be exactly balanced by in-sail forces. If light were completely absorbed rather than reflected, I think that we would get a shape known as a *catenary*, which is the form taken up by a hanging chain (where gravity acts on each link of the chain which has to be entirely supported by along-chain forces). This would not be quite right for a reflecting surface because the forces on each element of the sail would not then all be in the same direction. It would, however, probably not be too far away from the catenary shape and this might be a reasonable approximation for scoping calculations.