Measuring the Distance to the Moon by Parallax

Dr Michael McEllin   28 Sept 2018

The basis of the method is taking photographs of the moon at the exactly the same time from two points separated by a substantial difference. This experiment has been performed by partner schools in the UK and Canada. Even better would be a joint experiment between UK and African schools (because both locations are in similar time zones, so both experience night at about the same time). You can use a free "planetarium" software package such as *Stellarium*[1] to work out a suitable time for doing the observations.

- The experiment should take place at full moon.
- Use a small telescope or binoculars that can show the moon and also the star field surrounding it. You will need to fix these to a steady mount to make it easy to capture photographs.
- Photograph the moon at the same time. (Probably attempt a series of photographs - say once every 5-10 minutes - or maybe try to coordinate observations with live Skype connection.)
- After both sites have successfully captured pictures, exchange them by email and then use software such as *Astrometrica*[2] to measure the position of the moon accurately. (It is also possible to do this just by using Photoshop (or GIMP[3]) to superimpose the star fields and then measure the angular displacement in terms of moon diameters.)
- Use basic trigonometry to work out the distance.
- Older students should be capable of using the geographical coordinates of the two locations to deduce the correct baseline for the parallax calculation. I have, however, produces an additional document explaining the calculation, and also a spreadsheet which implements the algorithm.

Potential difficulties:

- You need access to mounted small telescope/binoculars. (50mm objective lens diameter should be all that is necessary.)
- Taking photographs. A friend of mine managed to take some reasonable astronomical photographs using his mobile phone camera - literally just holding it to the eyepiece -

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1 https://stellarium.org Free planetarium software capable of simulating views of the sky at any time and location on the Earth.

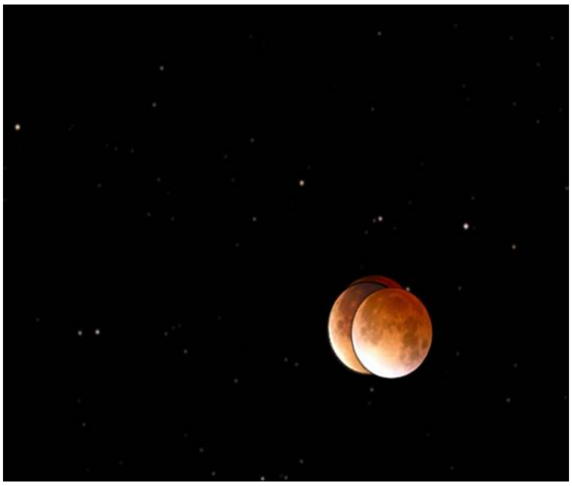
2 http://www.astrometrica.at Astrometrica is free for educational use. It is, however, really intended for use by serious astronomers and it takes a little while to learn how to use it fluently. An astronomical mentor with some experience of the tool would be very useful.

3 http://gimp.org GIMP has very extensive photo manipulation capabilities, though perhaps not quite as well packaged as Photoshop - but it is free!

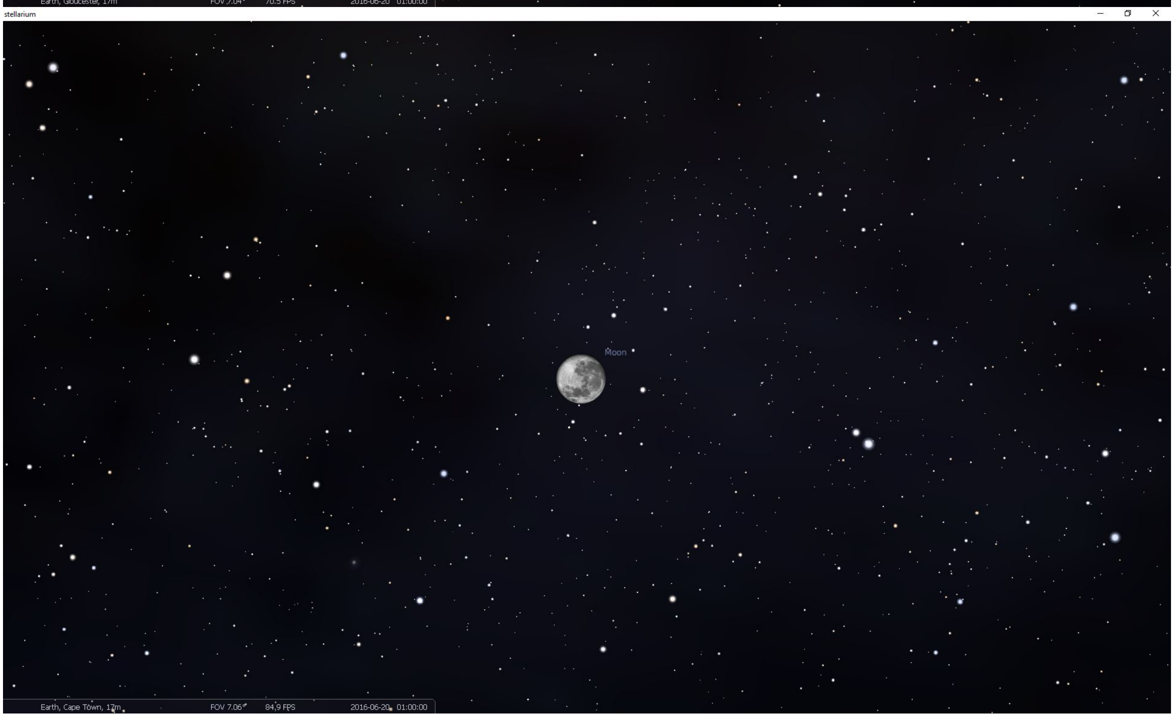
just to demonstrate that it could be done. It is not ideal but it will probably work well enough. Compact cameras or preferably SLRs would work better because they have bigger CCD sensors and are more sensitive to light and more stars show up. Ideally, the instruments at each site should be similar, though it is probably not essential. There is scope for a bit of prototyping and experimentation by the project team. See Appendix A for some advice on astrophotography with smartphones.

- Cloud cover at the wrong time. This is more likely in the UK, so would probably need to schedule several observing sessions. You could use something like Skype to coordinate observations.
- You need a late evening observing session with the students, i.e. cooperation with parents. Not everyone on the project team needs to be at the observing session. One could have a wider group of students doing the photographic measuring and final calculation.
- The team will need to select a good time to make the observations when the Moon is close to several bright stars that are visible both from the UK *and* from SA. (Work with *Stellarium* or one of the Moon phase websites (e.g. http://www.timeanddate.com/moon/) to look at the Moon's future track and find suitable periods. This will be an interesting learning exercise in itself.)
    - o Since we need it to be working after sunset in order to see stars and not have to observe in the middle of the night, Autumn to Spring is likely to be the best time of year
    - o It would be best if we had a full Moon – but all we really need is that the Moon shows at least one highly recognisable (say centre of a crater) that can be taken as the reference point for measurement.

It should all be doable, and enough challenges left for the students to get their teeth into (e.g. experimental details, measuring, errors etc.)



I have also attached a couple of images containing real photographs taken by schools. (This was back in 2009 when there was a "Year of the Moon". The original material has now been removed from the Web. However, a school in Maldon, Esssex managed to make several school link-ups: Maldon/Scotland, Maldon/Belgium, Maldon/Colorado and Maldon/Canada. I only include two results photos.) The image on the left superimposes photographs taken from Malden, Edinburgh and Belgium. The image on the right compares photographs from Malden, UK and Canada.



I have also used *Stellarium*[4] software to simulate a moon view from Gloucester in the UK and from Cape Town in South Africa, at the same time. (See the two images on the next page: the top image is the sky seen from Gloucester and the bottom that from Cape Town.)

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[4] https://stellarium.org Free planetarium software capable of simulating views of the sky at any time and location on the Earth.

Notice that the sky is in a different orientation in the two photographs. (The star background really do correspond, but you have to rotate the images carefully, as well as shifting them horizontally, to get a complete match.)

When you have satisfactory photographs it should be possible to make a first approximation calculation just by seeing how far the moon is displaced relative to its own diameter (which we know is approximately 0.5 deg). This is not going to be wildly out given that we should be looking at a displacement of at least one Moon diameter. There is a potential for a good discussion of experimental measurement error here, with several sources of uncertainty. (E.g. the Moon is not always at the same distance from the Earth so its angular diameter changes by a small but measurable amount over the orbital period.)

Much more accurate measurements are possible with *Astrometrica*, which is specifically designed to make astronomical position measurements. You will, however, need several stars in the field to will remain visible even when the Moon is spreading its light around, because the software needs to match a unique pattern of stars on the sky with the star catalogues to precisely locate every point on the image. Although this will be more accurate, it will also take *much* more effort - a good lesson in itself!

- I think that it would probably be a good idea to do some observation trials first in which one attempted to take photographs of the chosen star field *without* the Moon present. This might answer questions about the experimental method, such as will the camera be sensitive enough. How do you hold it steady against the eyepiece etc. How much control can one have over exposure?) We have to recognise that it might turn out that the camera is unsuitable for use with a very small optical instrument, such as 40mm/50mm binoculars - but we won't know unless we try. There might be a bit of sealing-wax-and-string manufacturing for holding stuff in the right position but I think that can be part of the real fun of doing experiments. (You ought to see some of the stuff in the Cavendish museum at Cambridge constructed in the grand tradition - how *did* they manage to get good results?)
- Later on one will need to work out if the camera can handle the contrast between a bright Moon and the stars. This tends to be where better cameras are likely to win out. An SLR will certainly work. A decent compact probably. An iPhone may have a good chance, but at the bottom end of the camera phone range it may struggle.
- *Astrometrica* has to be given information about the instruments being used (telescope/camera combination) - essentially telling it the angular extent of a photographic field. (otherwise it has too many possibilities when it is try to match star patterns). This might require some trial and error that would be easier with photographs of easily identifiable star clusters (Orion? Pleiades?). This is where a mentor might have to provide some closer support. It is easy once you have worked out how to do it, but not so easy to find your way first time.

I think that there are also simple experiments one can do in class. The difficulty, of course, is measuring relatively small angles, but see Appendix B for a suggestion. The subtlety with astronomical parallax measurements is that we have already, in effect, laid out an angular scale on the sky because astronomers have been measuring the positions of stars with respect to a standard reference coordinates for hundreds (if not thousands) of years. That is what makes it easier to do the job accurately.

See also these examples:

http://www.europeesplatform.nl/eloseducation/projects/measering-the-distance-to-the-moon/
http://www.astro.gla.ac.uk/users/martin/sss/autumn_distance.pdf
http://www.astro.washington.edu/courses/astro211/CoursePack/cp03a_distance_sun.pdf

Appendix A: Astrophotography with Smart Phones

a brief search of the Web turned up the following useful links:

- http://nightskyinfocus.com/2014/09/03/diy-phone-camera-to-telescope-adapter/
- http://astroblogger.blogspot.co.uk/2007/07/stupid-mobile-phone-tricks.html
- https://canadianastronomy.wordpress.com/2013/03/04/smartphone-astrophotography-how-to-photograph-the-moon-planets-with-your-phone/
- http://www.telescope.com/Articles/Equipment/Accessories/Whats-Hot-Astrophotography-With-Your-Cell-Phone/pc/9/c/192/sc/196/p/102897.uts
- http://www.universetoday.com/118527/iphone-astrophotography-how-to-take-amazing-images-of-the-sky-with-your-smartphone-tonight/
- http://www.lonelyspeck.com/photographing-the-milky-way-with-a-smartphone/
- http://iphonephotographyschool.com/night-sky/
- http://www.skyandtelescope.com/astronomy-equipment/universal-smartphone-eyepiece-adapter/
- http://www.iastrophotography.com/a-new-favorite-smartphone-adapter-for-astrophotography/
- http://shreddedpost.com/want-to-master-the-art-of-smartphone-astrophotography/

It also appears that apps are available for both iPhone and Android phones that allow more control over exposure times with the camera, allowing longer exposure at the telescope.

The consensus seems to be that smart-phone astrophotography is entirely feasible, but one needs some sort of adapter to hold the phone steady against the eyepiece. There are commercial devices available, but it looks to me as though one might be able to reproduce something similar quite easily in a home/school workshop. (Help from craft teacher?) Perhaps shape from discarded polystyrene packing? I am pretty sure that I could manage it in my own workshop if all else fails, but could be part of the challenge for the kids - how to jury-rig experimental kit from what you have lying around. (What every experimental scientist *really* needs to know!) I used to love this when I was in my early teens, but abandoned it for theoretical physics.

Appendix B: Local Parallax Experiment

I think that I have a decent experiment on parallax that sticks close to the astronomical principles.

Equipment: any type of camera, including smart phone cameral with reasonable resolution.
Protocol:

1. Go out onto your sports field. (You need to be able to see a distant view.)
2. You need a reasonably distinctive object on the other side of the sports field (e.g. not close, but at a distance small compared to the distant object).
3. Take two photographs with the same camera of the object at either end of a measured baseline (say 3 or 4 meters separation).
    1. Ideally use a tripod to keep the camera steady and level and at the same height. This is not essential, but it makes the subsequent analysis easier.
4. Also using the same camera, take a photograph of a tape measure or meter rule at a known distance. This will allow us the calibrate the camera so that we can say that each pixel corresponds to an exact angular increment.
    1. Use a photographic manipulation program (I used GIMP - but Photoshop probably does just as well) to look at the image of the ruler and count the number of pixels used to display a meter width. (GIMP tells you the position of the cursor in terms of pixel coordinates, so it is not real counting. Photoshop probably has a similar mode.)
    2. Use simple trig. to convert the horizontal distance represented by one pixel on the ruler into an angle.
    3. I estimate it should be possible to measure angular separations to an accuracy of about 1 minute of arc or better.
5. Now the slightly tricky bit.
    1. Using the two photographs taken on the sports field, import both into your image manipulation program as separate "layers". Make the upper layer partially transparent and move it around to align the position of the Roborough Fort between the two layers. (The Fort is playing the role of distant stars.)
    2. The "distinctive" object on the sport field will be out of alignment in the two layers and it will be possible to measure the displacement in terms of pixels, convert this to an angle and then use trig. to work out its distance.
    3. Check accuracy by measuring the distance on the ground. (You probably know the right distance along a touch line, for example, without measuring.)

This seems to me to be a reasonable analogy of the Moon measurement, and I just tried it out and it took about an hour to go through the whole process.

I went through this process from one of my bedroom windows.

Equipment

- Camera (I used a Lumix FZ18 - 8.1 Megapixels, 3264 x 2448 pixels horizontally x vertically.)
- Tripod
- Bedroom window with a fairly distant view. (Most distant horizon about 3 or 4 times the distance to the furthest object being measured - not really far enough but best that could be done.)

Method

- Attach camera to tripod and point towards a selected direction. (I tightened all the tripod screws and aligned two of the legs against the window frame.)
- Take photograph at left side of the window, one at the right side. (Taking reasonable care that the camera points in the same direction – but absolute accuracy is not essential since we will align the photographs with an overlay in a photographic manipulation program.)
    - Measure the **baseline** accurately.
- Load both photographs into GIMP (www.gimp.org – open source software similar to Photoshop).
- For photograph A, use GIMP to create a new "layer" and paste photograph B into the new layer.
- Set the opacity of layer B to 50%.
- Use the GIMP "move" tool to shift layer B until the trees on the distant horizon overlay exactly. (Zoom in on the photo below to see the effect. In fact, the day was rather dull and a bit misty, and the tree line has few really distinctive reference points so it was not very easy and may be slightly wrong.)
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- Use the GIMP cursor to measure the displacements in pixels of the object for which parallax displacements are to be determined. (The coordinates of the cursor in pixels are shown at the bottom left of the screen with GIMP. Photoshop may display this information in a different way.)
- In my image a middle distance fencepost was displaced by 30 pixels, while a more distance house apex was displaced by only 7 pixels. (You really need to zoom in to see this.)

Calibration of the Image

- Place a meter rule horizontally against a wall scale facing a camera. Ensure that the camera sees the full extent of the rule and that the scale is visible in the camera image.
- Take a photograph. (N.B. The French dictionary in the image above is just being used to wedge to rule! It has no astronomical function.)
- Measure the distance of the lens from the rule. (Some small ambiguity here? Is it the distance to front of lens or the focal plane that matters? Not an issue for smart phones – it may introduce about 5% error for my camera?)
- Use trigonometry to work out the angle subtended by the rule at the lens.
- Load the image to GIMP and use the cursor to work out how many pixels correspond to a meter (i.e. to the angle subtended by the lens)
- Divide subtended angle by number of pixels to work out angle per pixel. (Keep in radians to simply eventual calculation. However, should correspond to perhaps 1 minute of arc.)

Final Calculation of Parallax

- Parallax-angle = baseline/distance (angles are small so tan(x) = x, near enough if x is still in radians). Hence,

Distance = baseline/parallax-angle

= baseline/(pixel-displacement*radians-per-pixel)

My data is show below.

Width of reference rule (m)		1.00	meters		
Distance from lens to the standard rule		0.85	meters		
Angle subtended =		1.06345	radians	60.93	deg
Position of 100cm mark (pixels)	1	3127.00	pixels		
Position of 0cm mark (pixels)	0	72.00	pixels		
Width of rule in pixels		3055.00	pixels		
delta-angle/pixel		0.0003481	radians/ pixel	0.020	deg/pixel
Check with central region of rule:					
Position of 60cm mark (pixels)	0.6	1852.00	pixels		
Position of 30cm mark (pixels)	0.3	972.00	pixels		
Distance between 30 and 60cm marks		880.00	pixels		
		0.0003409	radians/ pixel	0.020	deg/pixel
Baseline for photographs		1.35	Meters apart		
Fence post parallax (pixels)		30	pixels		
Fence post angular displacement		0.0102	radians	0.59	degre es
Parallax distance estimate		**132.00**	meters		
Actual distance on Google Maps		**120.00**	meters		
Parallax of house roof apex		7.00	pixels		
		0.002386	radians	0.14	degre es
Parallax distance estimate		**565.71**	meters		
Actual distance on Google Maps		**350.00**	meters		

My estimate of the distance to the fence post looks OK (i.e. within about 10%). Not bad for a distance about 100 times the 1.35m baseline.

The estimate of the distance to the house is rather poor. The error looks too large to be accounted for just by measurement error. My guess is that my alignment of the two photographs was not quite correct. (The horizon probably was not really far enough away so aligning on the features at this plane probably meant that the two camera views were not properly parallel. This would tend to cause overestimation of the distances, particularly the longer distances.)

This is essentially the same method as that used for measuring the distance to the Moon – except that the distant stars really are very, very distant, so aligning photographs using these as a reference will certainly make the directions of the views parallel to a very good approximation. We might expect the move to be displaced by somewhat more than its own diameter if one of the base stations is in the UK and the other in South Africa. That is, between 0.5 and 1 degree.

N.B. when determining the parallax baseline for the real Moon-distance experiment, remember that it is NOT the distance along the surface of the Earth (a geodesic) but the real straight-line distance between the two points *through* the Earth. Unlike the other calculations for this experiment, this is an A-level grade maths problem, using polar coordinates (latitudes and longitudes) and spherical trigonometry. Students can just be given this distance.