Relativity for Cosmic Ray Project Members

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Cosmic rays travel very close to the speed of light, and you cannot understand the way they behave unless you know a bit about Einstein's Theory of Relativity. *Don't Panic!* We do not need to know very much and we do not need any difficult maths.

If you want to know more, I suggest you read Brian Cox's "*Why does E=MC^2*".

There are actually two Theories of Relativity. The 1905 "Special Theory" is a simple but brilliant idea – the so-called *Principle of Relatively* - applied to uncomplicated situations. Einstein's original scientific paper called "*The Electrodynamics of Moving Bodies*" is so clearly written that all the mathematical derivations can readily be understood by first year physics undergraduates. We, however, only need some basic ideas and a few of the results from the Special Theory. We do not really need to understand any of the maths.

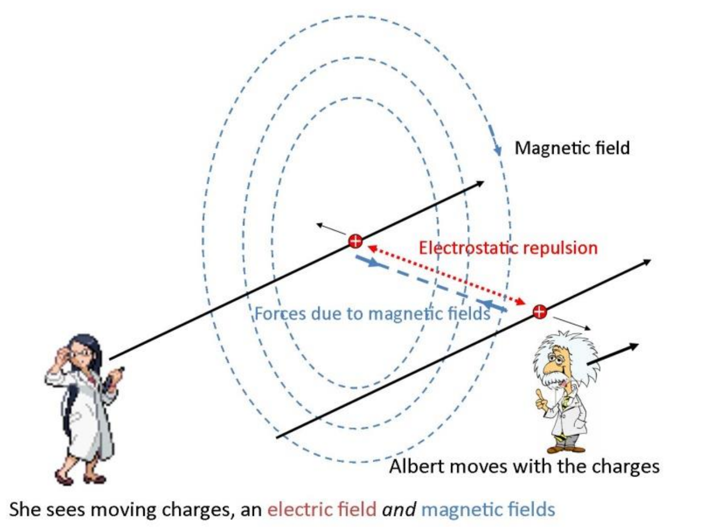
The 1915 "General Theory" applies an extended *Equivalence Principle* to situations where gravity is important and although the central idea is not complicated, and some deductions are straightforward, the full working out requires very difficult mathematics (even Einstein needed help from a mathematical colleague). Fortunately, fascinating though it is (black holes etc.), we do not need to know anything about General Relatively. Popular myths would have it that "Relatively is very advanced physics and really hard!" but that is only true about General Relatively.

Incidentally, "*Theory of Relativity*" is not a very good name, and Einstein did not use this phrase at all until about 1915, when others had already been using it for several years (though he did talk about the "*relativity of lengths and times*" and the "*Principle of Relativity*"). Relativity is really a theory of space-time *symmetry* because it says that every observer (read physics experimenter) moving at a steady velocity and well away from all gravitating bodies should be able to deduce *exactly* the same physical laws. This is Einstein's *Principle of Relatively*.

In fact, this *Principle of Relativity* is only an extension of ideas that everyone took for granted: no one would have expressed surprise at, for example, the idea that the laws of physics are the same at every point in space, or that they are the same whichever way you happen to be facing, or that they will be the same tomorrow as they were yesterday. (Well, doing experiments on a rotating planet it does at first sight make some difference, but we know we are on a rotating body and can take account of that.) 19th Century scientists also knew that Newton's Laws worked the same for two experimenters moving relative to each other at a steady velocity. Einstein's *Principle of Relativity* says that this symmetry is true for *every* physical law, including electricity and magnetism, and that turns out to have big implications. It was his genius to recognise that[1].

[1] *Warning!* Advanced Stuff! (But interesting.) In fact it was a German mathematician called **Emily Noether** who pointed out in 1915 that the invariance of physical laws over time and space was an important *observed fact* of nature, not to be taken for granted, and that this observation had far-reaching *deducible* consequences, such as the conservation of energy and momentum. Emily Noether deserved to be a lot more famous than she was during her lifetime, but as a female jew growing up and working in Germany she faced many prejudices. Although she was undoubtedly one of the most important mathematicians of her generation, when at first she gave lectures they had to be advertised under the name of a male colleague and she was not allowed to have an established professorship for many years, from which she was expelled by the Nazis. She began to receive the recognition she deserved after moving to the USA, but soon after died of cancer. *Neother's Theorum* is a foundation for a lot of modern high energy physics theory, so *we* know and can cheer her!

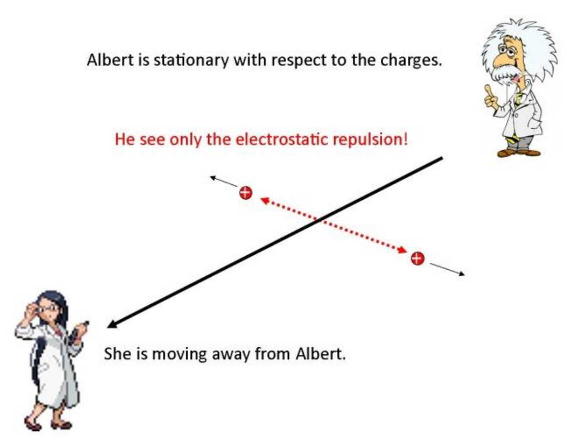
Most important breakthroughs in science start from a puzzle and Einstein's conundrum was that certain electricity and magnetism "thought experiments" seemed to require two different explanations for the same physical interaction. Two different physicists could watch the same experiment from different viewpoints and, with 19th Century understanding, would need to explain it in two different ways using different physical laws. (Einstein was a master of "thought experiments" some of which you could never actually do in a laboratory, but which perfectly highlighted problems of explanation in such a way that the right interpretation became so obvious that you did not actually need to do the experiment to see what would happen and how it should be explained.) Einstein said that the true laws of nature *had* to be such that every observer would agree on the same explanation. Modern physicists also now think that this is obvious, but no one had said it quite so explicitly before Einstein. OK fasten seat belts! You are about to do some university physics and if you understand the next few paragraphs you have understood the most important idea in relativity.



What is the experiment he was thinking about and what is the difference between our two physicists? Observer A is watching two electric charges moving through space side by side. They interact, in her view, firstly though electrostatic forces (perhaps they get pushed apart). In addition, because they are moving, they also constitute two electric currents each generating a magnetic field. Hence, in order to fully explain the subsequent motions we also have to add the force produced by a magnetic field on a moving charge. In this case, two identical charges moving in the same direction will tend to move together because of the magnetic fields.

Observer B, however, is moving along with the charges, so any motion he perceived must be entirely explained by the electrostatic interaction. Surely the two observers *must* see different accelerations!

How can this be? This is exactly the same situation interpreted in two different apparently incompatible ways.



Furthermore, we could also leave Observer A with two electric charges that start stationary with respect to her, but appear to be moving when looked at by Observer B. Should not the situation appear to be entirely symmetric? Albert should surely interpret the situation as requiring both the electrostatic and magnetic forces.

It is worth noting that from both viewpoints the observers only see the charges responding to forces that are directly along the line joining the charges, though they attribute these forces to electrostatic or the combination of electrostatic and magnetic forces. It is also worth remembering that the interpretation of the forces on charges in terms of electrostatic and magnetic fields is entirely dependent upon observations of the way charges accelerate – we have no way of directly observing fields. All we *see* are the motions: the fields are a mathematical interpretation and the electrostatic and magnetic fields seemed to have a reality because different observers would agree on the same interpretation. Einstein, however, pointed out that they do not agree!

The only way out within 19th Century physics would be to assume that there is some absolute standard of rest (and of course an absolute standard of time) and that the particles really do behave in different ways if they move differently relative to this fixed background. It seemed obvious to them.

In fact, up until that point physicists *had* assumed that electromagnetic interactions would be mediated by a universal, stationary “luminiferous aether,” a background substance that light waves travelled through, just as sound waves moved through air, and which also supported electric and magnetic fields. So, if you were moving in the same direction as the light wave you should see it moving a bit more slowly. If it were approaching, you would see it moving more quickly.

However, highly sensitive experiments carried out by Michelson and Morley had already failed to measure this effect in situations where it should have revealed itself. These experiments had shown that the speed of light appeared to be constant however you measured it. We now have enough evidence to accept this as an observed fact, measured to a very high level of precision.

19th Century physicists before Einstein were trying to account for this result with increasingly contrived explanations that involved, for example, atoms moving through the aether going slightly oval shaped so that all measuring devices shrank in the direction of travel.

Einstein said “one interaction, one explanation” and that if the two observers disagree about the speed of separation/closure of the two charges it is because *time and distance are different for each observer*! There is only one electromagnetic interaction, but different observers moving at different relative speeds will see charges moving in different ways because their clocks and rulers cannot be directly compared. It is only when you mistakenly insist that all standard rulers are always the same length and all standard clocks run at the same rate that you have to invent two different forces.

Einstein postulated that it was a *fundamental principle of nature* that every observer should see *the same speed of light*. We now have good reasons for thinking that this has to be exact because, for example, assuming that the speed of light is always *exactly* constant actually makes the laws of physics look much simpler (though this is the type of “simpler” that sometimes needs quite advanced maths to describe the underlying unifying patterns of the physical laws). The alternative is an increasingly baroque set of explanations involving complicated interactions with an invisible medium which is nevertheless always suspiciously undetectable. Quite a lot of modern physics does not quite make sense if the speed of light varies for different observers so this principle is now built into the foundations.

Ever since Einstein physicists have looked for other symmetries in nature – it is our most important guiding principle – the assumption that the laws of physics must always look the same from different symmetrical viewpoints. It has guided the way to modern theories of matter, such as the so-called "standard model" of particle physics. The next job for the Large Hadron Collider is looking for signs of a final potential symmetry, known as "Super-Symmetry". (Sometimes we do not find symmetry where we expected to find it. Strangely, and weirdly, we now know that the Universe reflected in a mirror would work differently. Lewis Carrol was right! The Looking-Glass World is different! Super-Symmetry would tie up all these loose ends.)

All Einstein's predictions turn out to be exactly right and we can now measure the effects directly, so we do not need to give any further discussion to alternative obsolete explanations.

Working out the equations of special relativity is not difficult; most of it just requires Pythagoras, a bit of mathematical stamina, and another even simpler thought experiment that considers how a flash of light or a radar pulse travelling between our two observers would be interpreted by each (who now know that it must be seen to move at the same constant speed for both of them).

This is normally taught at first-year university level and cosmic ray physicists really only need to understand something about the conclusions, but someone in the project did ask me to explain it, so I have put a derivation of time-dilation in an appendix. By all means skip it – but it was asked for.

The important results you need to know about are:

- Moving clocks run more slowly. The effect is very small in everyday life, but large when things (such as cosmic rays) move close to the speed of light. For our purposes the most important lesson is that rapidly moving radioactive particles appear to decay more slowly than if they were more or less at rest. This means that muons created in the Earth's upper atmosphere by a collision between a proton and an atom in the air, and which appear to decay very quickly if you are riding along with them, can actually reach the Earth's surface before decaying. We think that their clock is running very slowly. The effect is known as *time-dilation*.
- Moving objects get heavier. The effect is very small in everyday life, but large when things move close to the speed of light. As they approach the speed of light, when we try to accelerate them further, the work we do goes into increasing their mass rather than their speed. We see this effect all the time in our particle accelerators because we have to use increasingly strong magnetic fields to guide particle beams as they get more energetic.
- Moving objects contract in their direction of travel. If you watch an upper atmosphere collision while riding along with a cosmic ray, the new particles created spread out in all directions. However, the whole bunch is still moving towards the Earth's surface at nearly the speed of light so from our viewpoint it is heavily contracted in the direction of travel and looks like a flat plate. This is really important for measuring the direction of arrival of cosmic rays, because we can measure the exact time differences in arrival times at nearby detectors and know that in each case we are seeing the arrival time of the flat front of the shower.

In all these cases the factor, conventionally known as $\boldsymbol{\gamma}$ (gamma), by which clocks run more slowly, things get heavier or shorter is:

$$\gamma = 1/\sqrt{1 - {v^2}/{c^2}}$$

where **v** is velocity and **c** is the speed of light. So, when my clock has move forwards t_o seconds I would think that a moving clock will show a time advance of only:

$$t = \gamma t_o = t_o\sqrt{1 - {v^2}/{c^2}}$$

If v is 99.9% of the speed of light, for example, my clock will have clicked one second and I would need to wait another 19 seconds to see the moving clock tick the first second away. Similarly the mass of the moving particle would seem to increase by a factor of about 20 and its length (if we can measure it) decreases to just less than 5% or its resting value. Cosmic rays can be moving so close to the speed of light that the γ factor can be many thousands.

We now have clock so accurate that we can measure this *time-dilation* effect even when flying in commercial aircraft, and GPS tracking would not work at all if we did not take account of the time-dilation effect on the clocks in the moving satellites in orbit above the Earth. We could not get particle accelerators to work if we did not fully take account of the mass increase as we put more energy into the beam. HiSPARC can only work out arrival directions of cosmic rays because of length contraction.

People sometimes struggle with relativity when they come across an apparent (only apparent!) paradox: if you are rushing away from me in a high speed rocket, I think that your clock is running more slowly. However, from your view point, I appear to be rushing away from you, and you think that *my* clock is running more slowly. This would certainly be a real paradox if there were some absolute standard of time in the universe against which every other clock is compared. The solution explained by Einstein is that *there is no absolute time* – the concept simply does not make sense. Your time is your time and my time is my time. We can only compare times when our clocks are at the same point in space. When you work through a complete experimental scenario, remembering that you can only compare clocks at the same point in space, all the apparently paradoxical elements vanish, and every observer agrees about what is actually going on. (You will do this exercise in most first year university physics courses.)

So, that is the way it is: everyday assumptions about time are just wrong and *absolute time does not exist*. Get comfortable with this concept if you want to do cutting edge physics!

In fact, many of the equations derived by Einstein were written down first by other people, and physicists still talk about "Fitzgerald-Lorenz" contractions and "Lorenz" and "Poincare" transformations. (These were derivations produced to fix up 19th Century physics with oval atoms and so on.) They, however, did not make the big step in interpretation – abandoning absolute time. That is why we credit Einstein for completely changing our view of the world.

There is a better, more modern way of looking at all this which just says that all distances in space-time must be the same for all observers, provided you measure "distance in space-time" the right way. We can start to understand this by thinking about distance between two points in space (we are back in ordinary space now – not space-time). The distance is actually the same whether we choose to measure it in inches or centimetres. It is also the same whether we choose to lay a ruler directly between the two points in the shortest line, or we lay three rulers at right angles to each other in some other directions and use Pythagoras to work out the distance between two sets of x,y,z coordinates defining the two points. The distance is a real property of space independent of the measurement. We say that distance is an *invariant* property of 3D space.

We now know that that method of measuring distance is not invariant (though only very, very slightly wrong in the everyday world where motions are mostly very slow compared to light). The right way to produce an invariant measure of distance in space-time, that all observers will agree with however they measure space and time, is to use $\sqrt{x^2 - c^2t^2}$ where x is the space separation and t is the time separation and c the speed of light. (This idea was invented by a mathematician called Minkowski. Einstein hated this "unnecessary" viewpoint at first, until he realised it was the only way to handle calculations involving gravity.)

Why is the World like this? We could say: "*Well that* <u>*is*</u> *just the way the World is made*!" We can perhaps see slightly further than that if we demand that *causality* has to be same for all observers: if A *causes* B then A has to happen *before* B whoever you are and from wherever you are observing. It turns out (after some sophisticated maths) that there are only two ways to build a causally consistent physics: one of them (the simpler one) is Einstein's. The other (the complicated way, which, fortunately, we now know does not work) is the 19^{th} Century way. Einstein's explanation has a universal maximum speed limit which in the theory is the speed of special type of particle, which can carry energy and momentum, but has no mass. The speed of such a particle can never change in this theory. The light photon appears to have all the characteristics of such a massless particle, and it is the fact that the photon is massless that gives light its special status in the Theory of Relativity.

So, when we say that nothing can travel faster than light, we are really saying that nothing can travel faster than a massless particle, or in a sense, no faster than causality.

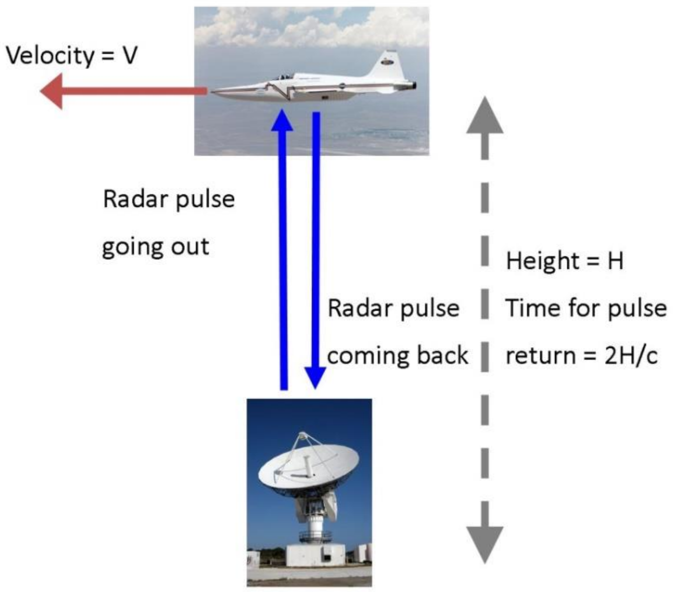
Well, that *is* just the way it is as far as we know. I do admit, however, that my assertion that space-time physics is simpler than space physics *with* time will not look convincing until you find a need to perform real calculations involving high energy particles.

How does the world look from a cosmic ray (say a proton) moving at 99.999% of the speed of light? One of the main differences is that any light or radio waves coming towards the particle will appear to be shifted to much higher frequencies and carry much larger amounts of energy. So a light photon from a star might now look like a high energy gamma ray photon and may knock the proton about somewhat when it bounces off it. In particular, for the very, very highest energy cosmic rays even the universal microwave background (very low energy photons) looks like a stream of high energy gamma rays in the rest frame of the proton and their collisions blead its forward motion away. This is an argument against such high energy rays coming from extra-galactic sources. They should just never get here! You might see references to this as the *GZK limit* (GZK standing for the names of three scientists: Greisen, Zatsepin and Kuzmin who worked out this theory). Any particle with an energy larger than 5×10^{19} eV (about 8 joules) cannot have come from anything further away than our relatively close neighbouring galaxies, and certainly not from events close to the Big Bang, for example.

At least one particle with an energy of about 3×10^{20} eV appears to have been detected (this is sometimes known as the "OMG particle" – Google it!) which is well above the GZK limit. This is so much higher than the energies we can achieve in Earth-based particle accelerators that some people wonder if new physics comes into play. These particles are very rare, but with lots of detectors all over Europe, it is possible that HiSPARC could be involved in finding more particles of this energy. This is absolutely cutting edge science.

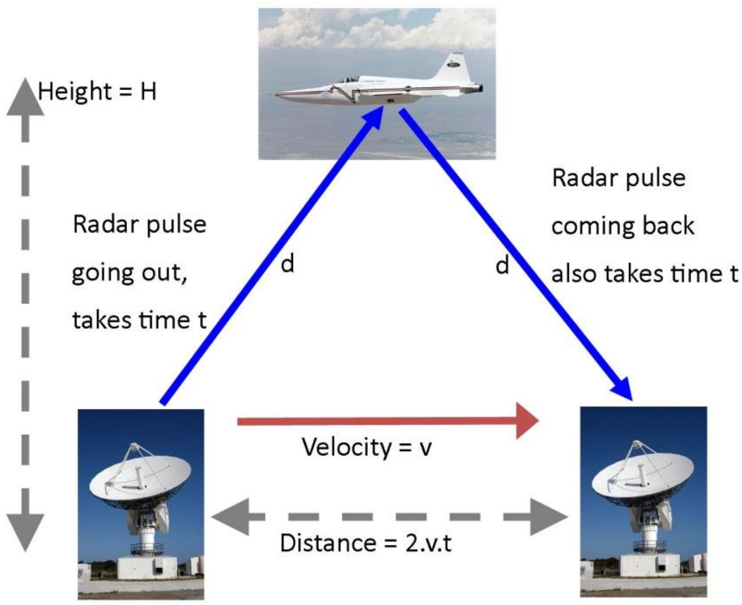
Appendix: Time Dilation

We are going to do another thought experiment. This time we will have a radar dish on the ground and a fast aircraft flying overhead. Remember, this is university level physics. It is not supposed to be easy to understand before then, but I was asked to explain it by one project member!



Radar pulses travel at the speed of light (which all physicists conventionally denote with the letter "c"), so it is easy to see, *from the viewpoint of the radar operator*, that the pulse will take a time H/c to reach the plane overhead and the same time to come back - in total 2.(H/c). (In this viewpoint the velocity of the plane is irrelevant.)

There is, however, another viewpoint, that of the pilot of the plane, who sees the radar station moving at velocity V in the opposite direction.



From the pilot's viewpoint each pulse has travelled a longer distance, *still at the same speed c*. The pilot will predict that it must take a longer time to do this. We could imagine that the radar station starts sending out its pulses just in time for the first to meet the plane overhead, then keeps sending out the pulses at regular intervals, say once every millisecond, and stops immediately a return single is detected. Hence, the pilot in the aircraft, who can detect all the pulses and count them, also knows exactly how many milliseconds were counted down on the ground for the return trip.

We can work through the pilot's prediction. Let us suppose that in his "frame of reference" (the conventional Relativity term) the whole round trip takes a time 2t (so t is the time for the pulse to reach the plane), then d = t.c is the distance along the hypotenuse (half the total there and return distance). We also know that the distance "travelled" by the radar station must be v.2t at the point where the pulse returns.

So, since $d^2 = H^2+(v.t)^2$ and we know that d = t.c, then we must have $c^2t^2 = H^2+v^2t^2$. Hence the pilot calculates that the pulse must have taken a time $2.t = 2.(H/c) / (1 - v^2/c^2)^{1/2}$. Remember, the pilot can see the pulses from the ground which are supposed to come at 1 millisecond intervals, counting up to 2.(H/c). The pilot has to conclude that the radar clock is running slow by a factor $\boldsymbol{\gamma} = 1/(1 - v^2/c^2)^{1/2}$.

Note that the plane also has its own radar looking down at the ground, and we can reverse the viewpoints. Now the radar operator would conclude that the plane's clock is *also running slow* by the same factor. What?! How can this be!?

For people who are raised in the implicit assumption that there is a sort of "true" master clock keeping time for the universe this looks like a paradox. It is not. *There is no universal time-keeper*. Time is something that belongs to each moving object in space and each clock runs at a different rate (though often only slightly different). You can only compare times of different clocks by exchanging light signals (or radar pulses) and the only way to know two clocks are synchronised (perhaps just for an instant) is when they are at the same point in space.

For typical everyday velocities (of planes, say 1000km/h) the factor is very small (about $1.5\text{x}10^{-9}$ seconds in an hour). This is nevertheless detectable with modern atomic clocks and the experiment has been done. Einstein is right. It is, in fact, an *essential* correction for the clocks in the GPS satellites – otherwise the calculated locations are in error by hundreds of meters. Some cosmic rays are travelling so close to the speed of light that they have gammas of 1000 or more, so particles that are created in the upper atmosphere and which should decay in millionths of a second last for milliseconds and reach the Earth's surface.

Incidentally, we cannot make particles travel faster than light: they just get heavier and heavier the more we push. All the energy is increasingly converted into mass ($E=MC^2$ etc.). Only massless particles, like a photon of light, can move at the speed of light, and they cannot slow down.

The main difficulty about Special Relativity is keeping your head straight while thinking about all the different viewpoints and how they compare, and finding the correct interpretation that avoids apparent paradoxes (there are none). The "gamma" factor, $\boldsymbol{\gamma} = 1/(1 - v^2/c^2)^{1/2}$, turns up all through relativity but you *will* need to study university physics if you *really* want to know why it makes physics simpler. There are lots of other deductions about mass increasing as we go faster, length contraction for moving rulers and so on. Most of the derivations are no more difficult than the one we just did, but learning to apply Relativity to a wide variety of problems is definitely university level.

You will have to get a book on Relativity of you want to learn more, such as Brian Cox's "*Why Does $E=MC^2$*".