Project: Design Study for a Space Elevator

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1 Motivation

The hardest and most expensive part of spaceflight is getting from the Earth's surface into orbit. (The second hardest is probably getting back down again!) This is an issue of *potential wells*: a kilogram orbiting in space has a lot more *potential energy* than one at the Earth's surface. It also has to keep moving in order to stay in orbit and that means that there is also *kinetic energy*.

Once we are well out of the Earth's potential well - say, somewhere like the Geostationary Orbit (GSO) height the energy cost of getting anywhere else in the Solar System is relatively low.

Furthermore, although we have to exert very high forces to lift a massive rocket against the acceleration due to the Earth's surface gravity and also to accelerate it to orbital velocity, once we are in orbit the forces required are much lower - providing we do not have to do anything very quickly.

At present, however, the energy still has to come from burning rocket fuel and all that fuel still has to be lifted from the Earth's surface. This process is very energy inefficient because we have to burn the rocket fuel all the way from the Earth's surface until we get into low-Earth orbit (LEO) , so the fuel that we burn later in the flight has itself to be lifted out of the gravitational potential well and also has to be given kinetic energy, as part of the rockets motion.

It turns out that for every kilogram of fuel we may want to burn after leaving GSO we probably have to burn about twenty kilograms or more getting it there.

If we want to have a serious industrial presence in space, for example to mine asteroids for rare and essential elements that are running out on Earth we need a better way—otherwise it will all get too expensive.

The fuel efficiency of a rocket motor depends on the amount of momentum you can give to each kilogram of material in the exhaust plume. If you can can eject the exhaust faster you use less fuel to get the same acceleration.

The exhaust velocities of chemical propellants are limited by the amount of chemical energy contained in each kilogram of fuel: the higher the specific energy content, the higher the potential exhaust velocity. At the atomic scale it is a question of releasing energy in a chemical reaction between two molecules which is then used to accelerate the reaction products. Ideally you want reactions that involve big releases of energy, but you also need light molecules so that they heated to high temperatures and accelerated to high velocities. This is one reason why NASA's space shuttle was partly powered by using liquid hydrogen and oxygen as the propellant: it has a high specific energy release and the lightest product molecule. (There are other reasons why engineers often choose *not* to employ this combination, not least because explosive cryogenic liquids are very difficult to handle safely.) All this means that our rocket motors have pretty much reach the theoretical limit of what is possible with chemical propulsion.

The *Tsiolkovsky Rocket Equation* nails down the fundamental relationships that dominates the thinking of every rocket engineer and astro-navitagor:

$$\Delta v = v_e \log\left(\frac{m_0}{m_f}\right) \quad \Longrightarrow \quad m_o = m_f.exp\left(\frac{\Delta v}{v_e}\right) \tag{1}$$

where:

- Δv is the change of velocity actually achieved,
- v_e is the exhaust velocity,
- m_0 is the initial mass (including propellant - the 'wet' mass),
- m_f is the final mass after the rocket shuts off. It is sometimes called the 'dry' or empty mass, but the equation still applies if you need to keep fuel in your spacecraft for further manoeuvres. The equation relates the '’before' and 'after' mass of the spacecraft to the velocity change.

This basically says that if we have to work with a fixed v_e then we are on a hiding to nothing as we try to attain higher and higher velocities (e.g. if we want to send a spacecraft out to Jupiter in a reasonably timescale). You are pouring more and more energy into accelerating more and more fuel just so you can throw it out of the back of the rocket.

This is why nearly all deep space missions use planetary 'sling-shots' to give spacecraft very high velocities. (The astro-navigators design the course so that the spacecraft effectively bounces off the potential well of an approaching planet - fundamentally the same as a using tennis racket to increase the velocity of a ball, except that the we do not actually hit the planet, just get pulled around the back in a close orbit.)

See Appendix A for a discussion of the energy cost of getting into geostationary orbit (which is our current standard of comparison). The appendix also illustrate the importance of the ΔV concept in astronavigation.

The other problem with our launch methods is that we end up throwing away most of the launch vehicle. A rocket booster is a very expensive piece of hardware and in most cases it is only used for a few minutes at the start of a flight and then falls back into the ocean. While the fuel required to get a few kilograms into orbit is expensive, the booster hardware is *extremely* expensive—and a lot of energy has been used to make it. I suggest that you research the actual costs of fuel and the costs of the hardware.

More recently companies such as SpaceX have demonstrated reusable boosters that fly back to their launch platforms. This does, indeed, substantially reduce the cost of launches, but there are downsides and limits to what is possible. Firstly, booster hardware—especially the rocket motors—is very highly stressed during its flight, so the second and subsequent flight are predicted to have higher risks of launch failure. Hence, if you have a really expensive and critical satellite to launch, you pay extra for first-use of the booster. There is also a significant amount of maintenance required between launches with deep inspections on the rocket motors, and replacement of critical parts. It is not at all like reusing a commercial aircraft on hundreds of flights each year between major maintenance inspections.

At present, the cost of doing thing is space is far too high to consider some potentially valuable applications, such as mining rare metals from asteroid. (We a rapidly mining out the ores of some metallic elements with uniquely irreplaceable properties, but we know that some asteroids have these available in their rocks in much higher concerntrations than we find near the Earth's surface.

2 Proposal: Investigate the Feasibility of Building a 'Space Elevator'

The idea of a space elevator was brought to public attention by the visionary science fiction author, Sir Arthur C Clarke, in his late 1979 book "The Fountains of Paradise" but the idea has a much longer history with various discussions going back as 1895. The modern form of the idea, however, was studied by Jerome Pearson in 1969.

The basic idea is to have a large counterweight sitting in geostationary orbit (this might well be a large asteroid towed into position). From this we would drop a cable down to the Earth's surface.

Now, instead of burning rocket fuel to get into orbit, we have modules that grip the cable and climb out to the counterweight. From there it would be possible to travel around the Solar System at little energy cost because we are almost entirely out of the Earth's deep potential well. The climber modules would then return to Earth, carrying material (such as rare minerals) that we have either made or mined in space. In principle it might be possible to reduce the cost of getting to high Earth orbit by a substantial amount—maybe by a factor of 10—mainly because all the hardware should be highly reusable, and although we still expend energy climbing the cable, we might be able to recover much of it when the module descends (since we have to apply a braking force to stop it accelerating down at ever increasing velocity, it should be possible to make it do work).

If it was easy, we would have already done it. There are, in fact, a good many technical problems, but none of them are so far beyond our current technology that the prospect is hopeless.

Your challenge is to investigate the gap between aspiration and reality, by studying the physics of a space elevator and performing calculations which illustrate the problems and that identify the targets that need to be attained.

3 A Brief Survey of Issues

3.1 Strength of Materials

Our cable hangs down from the geostationary counterweight. That means that the cable up near the top is supporting the weight of all of the cable. Note that the force of gravity is, of course, falling off a $1/R^2$, so we have to do an integral to work out the tension in the cable at any point. This is, however, a difficult calculation.

We would soon find that a cable made out of something like high-tensile-strength steel would only get a fraction of the distance down to the Earth before its own weight caused it to snap. (I am going to leave it to you to look up the numbers and perform the calculation.)

We cannot improve the situation by making the cable thicker, of course. We might double its strength by doubling its cross section, but we also double its weight, so it still snaps at the same point.

It is possible to do rather better, however, if we taper the cable, so it is thick near the top, where it has to carry the full cable weight, but gets thinner as we go down.

It is not particularly hard to calculate the ideal variation in cable cross section that gives maximum length before snapping. (It is well within the scope of A-Level maths, though you will have to work at it.) We will still find, however, that materials like steel simply cannot be made strong enough to support their own weight. The fundamental issue here is that though fairly strong inter-atomic bonds bind together the atoms in steel alloys, iron is a heavy atom. There is little that can be done to increase the strength of the bond—they are limited by the chemistry and the properties of atoms. There will be a fundamental limit to the tensile-strength/weight ratio of any steel alloy[1]. We therefore need to ask whether there are other materials that potentially have higher strength-to-weight ratios. Glass has a high strength-to-weight ratio, but is also brittle, so is not a good engineering material. Glass reinforced plastic (GRP) sometimes called fibreglass overcomes some of the problems, but is still not good enough. Carbon fibre (again held in plastic resin) is even better, but still falls short.

Note that we are looking at light atoms, such as carbon, which still have strong inter-atomic bonds. Can we do better than carbon fibre? Maybe!

Recent work has investigated the properties of carbon nano-tubes and another form of carbon now called graphene. These may well meet the strength-to-weight criteria, but they have only so far been made in very small quantities, and no one knows whether it would be possible to manufacture them into large, sufficiently defect free engineering structures. We can speculate, however, and this is what I would challenge you to do. What would become possible if we did achieve the ultimate theoretical strength of these exotic carbon-based materials?

The above problems, bye the way, may be limiting for a space elevator above the Earth, but it are not necessarily true for elevators above the Moon

[1] We need to point out that real alloys are often show much less strength than the theoretical limit. This is because real materials normally contain microscopic defects that concentrate stress just where you do not want it. Glass, for example, is actually a very strong material in tension, but if you scratch the surface one gets a concentration of stress at the tip of the scratch which can often easily exceed the tensile strength locally. So, a few atoms can be easily pulled apart by the applied forces, but this makes the scratch turn into a crack, and the more the crack grows the higher the stress concentration, so the faster it grows.

Metals such as steel, have a rather more complicated way in which crystal defects reduce their strength—and it also explains why alloys such as steel are much stronger than iron. Look up 'dislocations'.

or Mars, which have smaller gravitional fields. Even if we never do it for the Earth we might well still want to do it elsewhere at some point.

See Appendix B for further discussion of material strengths.

3.2 Energy Supply to the Climber

How do we feed power to the climber module, so it can ascend the cable?

One could try to give it an internal power source (e.g. a battery, or even rocket fuel). It is, however, very instructive to perform an integral, of the type used to derive the rocket equation, and work out how much internal fuel would be required to reach geostationary orbit (compared with the requirements of the type of orbits discussed in Appendix A). I would encourage you to do this—it reveals some important principles.

We would not go down this route just because it saves lots of fuel. (You find out what if any is the saving.) We mainly do it because it saves hardware.

We do, however, have with a cable the possibility of feeding energy to the climber modules from the outside: the do not have to carry their energy source with them.

A number of options can be considered (you should follow through!):

- Generate electricity from solar panels attached to the climber module. Obviously this is probably not viable in the Earth's atmosphere, because of the aerodynamic drag, but the panels might be extended once in space. Can we get enough energy this way? How big would the panels need to be? How heavy would they be?

- Could energy be fed from the cable? Would it be possible to place solar panels all along the cable? Could you embed super-conducting power cables in the elevator cable and feed energy from a massive solar array at line-top?

- To what extent could we extract energy from descending modules, and feed electricity across to climbing modules at about the same height? (Here we assume that there will be a more-or-less frequence sequence of modules climbing and descending.)

- Can we 'beam' energy down to climber modules from massive solar arrays at geostationary orbit. (There have been experiments in turning electricity into microwaves and beaming then to a distance antenna where it is turned back into electricity at surprisingly high efficiency.) Is this feasible? Could we use laser beams?

3.3 Constructibility

- Have you ever wondered how you put long bridges over wide gaps?
- How do you provide support during the construction process, when there is nothing in the middle?
- How do you erect the tall cranes that are used to build tall building?
- Do you need taller cranes - and how do you erect those?
- How would be actually build a space elevator?
- Where would the materials come from?
- How do you turn them into the form required?
- Where would the energy come from?
- How do we actually get it down to a particular point on the Earth's surface and safely fixed down without flailing around?
- Could you do this with human labour—or do you need robotic manufacturing techniques?

3.4 Safety Implications

What would happen if the cable snapped (perhaps after being hit by a meteorite)?

3.5 Finance

How Much Would it all cost?

4 Do it with Numbers!

The essence of a science and engineering feasibility study is that you do the sums.

There is a great deal of speculative descriptive material on the Web. We want to know whether any of the science actually hangs together. The core skill of a practicing professional physicist is turning wishes and speculations into questions that can addressed in a quantitative way—even if the answer consists of phrases such as "probably better than" and "probably no worse than".

As with most physics, the questions can ultimately be addressed by thinking about energy, power, force and strength.

For those of you who are seriously thinking of degrees and subsequent careers in physics or engineering I strongly recommend the following two books, which are available to download in PDF format under the Creative Commons licence.

- *The Art of Insight in Science and Engineering* https://mitpress.mit.edu/books/art-insight-science-and-engineering
- *Street Fighting Mathematics* https://mitpress.mit.edu/books/street-fighting-mathematics

The above publisher's pages both contain an "Open Access" link for downloading the PDF—though just Googling the title is likely to bring up direct links to the PDFs. Both of these books teach the art of finding 'good-enough' answers without having to produce exact solutions to complicated problems, and show you how to understand the essential nature of problems, without loosing yourself in mathematical manipulations.

Appendix A: The Importance of Delta-V

The *Tsiolkovsky Rocket Equation* expresses the relationship between the increment in spacecraft velocity and the proportion of the fuel that is consumed achieving that change:

$$\Delta v = v_e \log\left(\frac{m_0}{m_f}\right) \qquad \Longrightarrow \qquad m_o = m_f.exp\Big(\frac{\Delta v}{v_e}\Big) \tag{2}$$

When using chemical propellants it is nearly always sensible to use fuel in short, high-thrust 'burns' because in that way we may be able to avoid carrying fuel out of potential wells.

So, having achieved a circular low-Earth orbit (at some cost in fuel), we may now wish to move to a higher orbit (for example, geostationary orbit). The normal way we might perform this manoeuvre would be to give a momentum kick to the spacecraft with a short period of high thrust that quickly increases its velocity. (We shall assume that this ΔV is acquired in an effectively instantaneous internval.) Since the spacecraft is now moving too fast to maintain the circular orbit it will now be moving in an elliptical orbit with the perigee (the point of closest approach) at the point where the change in velocity occurred. At some point later the spacecraft will be at apogee (the point furthest away from Earth) but having climbed out of the gravitational potential well for some distance is now moving more slowly. It is, of course, now moving too slow to maintain a circular orbit at the apogee distance so starts to 'fall-back' towards the Earth.

At the apogee, however, we can stabilise a circular orbit if we perform another burn to deliver just the right ΔV to increase the spacecraft velocity to that required for a circular orbit at this height. It is easy enough to work out the amount of fuel required for this change in orbits. If the first velocity increment is ΔV_1 and the initial mass is m_o while the mass after the first burn is m_1 then:

$$\Delta V_1 = v_e \log\left(\frac{m_0}{m_1}\right) \tag{3}$$

For the second burn we start with mass m_1 and then finish with mass m_2 having incremented velocity by ΔV_2 so,

$$\Delta V_2 = v_e \log\left(\frac{m_1}{m_2}\right) \tag{4}$$

So, the ratio of the initial mass to that after the second burn m_o/m_2 is just given by:

$$\Delta V_1 + \Delta V_2 = v_e\Big(\log\left(\frac{m_o}{m_1}\right) + \log\left(\frac{m_1}{m_2}\right)\Big) = v_e \log\left(\frac{m_o}{m_2}\right) \tag{5}$$

That is, in order to calculate total fuel requirements for several manoeuvres we just add the ΔVs for all the the individual steps, and then use the rocket equation (2)

Many astronavigation problems are described in terms of ΔV - e.g. "the ΔV required to rendezvous with asteroid XYZ".

It may be useful to go outline how one might calculate the required ΔVs for a simple situation. (You can follow through with the detail.) See Figure 1 for an illustration of the problem of transferring a spacecraft to a higher orbit (where both orbits are circular). We have two knowns: the radius of the low orbit, r_1 and the radius of the high orbit, r_2, both of which also determine the orbital velocities, V_1 and V_2 from Newton's law of gravitation.

Our task is to calculate the velocity changes required to enter and leave the elliptical transfer orbit. Note that we do *not* need to calculation the exact shape of the ellipse in this simple case because we can rely on two conservation laws: conservation of energy and conservation of angular momentum. The energy at the two burn-points is easy to calculation using the gravitation law as the sum of potential and kinetic energy, which are entirely determined by the position and the velocity. The angular momentum at the perigee and apogee of the ellipse are also easy to calculation because the velocity is exactly perpendicular to the the radius vector at these points. You should now be able to write down two conservation equations, which have two unknowns: dV_1 and dV_2 so you can solve for the unknowns. We then just add the two ΔVs in order to calculate the fuel requirement from equation 2.

Most of astro-navigation using chemical propellants is about gluing together sections of elliptical and circular orbits, connected by ΔV changes. It does, of course, get more complicated when cannot do our velocity changes at the apogee and perigee of the ellipses, but that is why we have computers to do the complicated detail for us.

In reality, when we want to go to Mars or Jupiter, which would required quite large ΔVs we tend to plan orbits that involve 'slingshots', which uses close orbits around the back of other planets to increase our velocity and energy for free.

ΔV do not help us with fuel calculations for continuous-thrust courses, which are usually less efficient than this quick-burn+coast option. However, while low-thrust propulsion cannon give the almost instantaneous impulses of chemical propellants they burn fuel at a much lower rate and use very much higher exhaust velocities, so they are viable options.

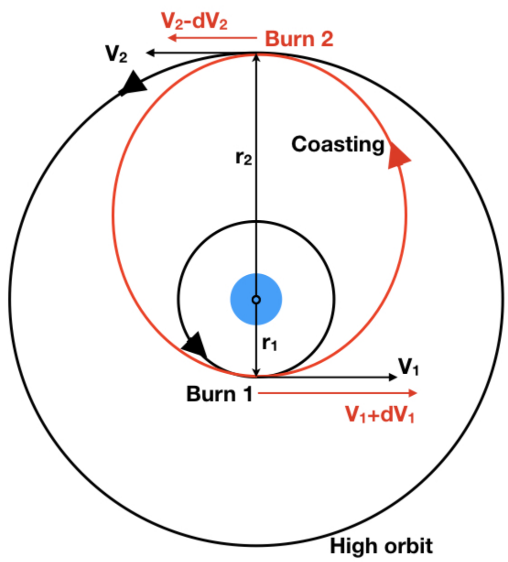

Figure 1: Transfer Orbit

Appendix B: Relevance of Strength of Materials

All the materials that we see around us are held together by atomic bonds which are ultimately based on electromagnetic forces. In principle, the maximum strength of a material depends on the strength of the atomic bonds holding it together. If we want to pull on, say, the two ends of a rod until it breaks in half we have to supply a force that overcomes the combined interatomic forces holding together the planes of atoms in the bar. Real materials always seem to be much weaker than the strength we calculate in this way because they nearly always contain small defects that tend to concentrate stress, so we do not have to break a complete plane of bonds all at once, instead we can break them one by one, like tearing a sheet of paper once a small nick has been made in one edge.

We are used to thinking of materials like steel as very strong, but they are also very dense. Iron is a heavy atom, but inter-atomic forces do not necessarily increase in proportion to an atoms mass. In fact, in terms of its strength/weight ratio, steel is not very good. Aluminium is, of course, a much lighter atom, but its bond strength is also much lower so although its alloys can have higher strength/weight ratios than steel (which is why they are used in aircraft construction) it does not provide a solution to our problem.

We need to look at light atoms that still have strong inter-atomic bonds, and down at the bottom end of the periodic table there are few options, which pretty much resolve down to exotic forms of carbon. We have used carbon fibres in aircraft construction for many years because of its very high strength/weight ratio (which has to be set in epoxy resin because it is otherwise too flexible if you want to build aircraft spars). You could, however, imagine weaving a material out of carbon fibres.

More recently there has been a lot of interest in materials such as carbon nano-tubes and graphene (which I will leave you to research). These promise to be much stronger than conventional carbon fibres and one might imaging using them to make very large and very light sails—if we could produce them in sufficient quantities. That at the moment is the technical barrier: in theory a film of graphene of about the thickness of cling-film could support an elephant, but the largest defect-free samples produced so far are finger-nail sized.